1951

Vector power distribution in electric networks

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VECTOR POWER DISTRIBUTION

IN

ELECTRIC NETWORKS

by

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A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of

The Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major Subject: Electrical Engineering

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

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Signature was redacted for privacy.

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1951

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I. INTRODUCTION

A. The Development of Power Systems

Electric energy has attained a prominent position among the several forms of energy which are commonly employed to perform tasks and supply services in our modern world. Much of the prominence of electric energy is due to the ease and economy by which it is transmitted over great distances with only a slight energy loss.

The means by which energy is transmitted between electric power stations and the locations where this electric power is utilized is called an electric power transmission network. The electric power transmission network together with the associated generators, loads and transforming equipment is spoken of as an electric power system.

The earliest formal electric power system was constructed by Thomas A. Edison in 1882. This power station, known as the Pearl street station, was constructed by Edison in New York City primarily for the purpose of providing direct-current electric energy for the operation of arc lamps and for the operation of his newly-invented incandescent lamps.

Edison's electric power system was immediately successful, and similar small electric power systems appeared very soon thereafter in cities all over the United States. These networks were soon found useful not only as a means of transmitting energy for illumination, but useful as well as a means of transmitting electric energy to motors which could perform a host of industrial tasks.

Within the short span of a half dozen years electric power companies expanded until they became confronted with some of the inherent difficulties of the direct-current power system. By 1886 the transformer developed by Stanley made it possible for George Westinghouse to develop the alternating-current power system with the advantages which accrue through transmission at very high potentials.

Further advantages in power transmission were realized soon after 1890 by the use of the three-phase system proposed by Tesla.

As the years went on the increased operating efficiencies of extremely large generating plants made apparent the advantages of extended power transmission systems serving large areas. Until about 1920 these power systems were of the "radial" type with a single generating plant providing power to lines which served near-by areas that were served by no other plant. By this time power systems had grown so large that in many places one system would border geographically upon another. This made it easily possible for such bordering systems to be connected together. Such interconnections proved to have considerable value from an operating standpoint. If a given generating plant were to fail or be unable to carry all of its load the companion generating plant of the other system could be called upon to maintain service. If a portion of a network should fail, it would often happen that the network could be at least partially restored to operation by energy coming from the second plant situated beyond the break. Thus interconnection has much to offer as a means of assuring continuity of service.

An interconnected system need only provide generating capacity adequate for the system peak load. This, in normal service, may be expected to be less than the sum of the peak loads of its component systems.

As time passed interconnections became more numerous and more complex. All interconnected systems may, however, be separated into two classes. These two classes are the loop, or ring, type of interconnection and the non-loop, or radial, system. Loop systems are systems which close upon themselves and hence allow the flow of circulating currents if the voltages at the point of closure of the loop, before closure, are not equal in both magnitude and phase. Such loop systems also include those systems which involve other systems as portions of the closed ring. The distinguishing feature of a loop network is that a point on such a loop network may receive current over more than one path from a single source. This gives rise to the possibility of the existence of circulating currents on the network. These circulating currents may be controlled by proper means and when this is done it will be found that a large measure of independent control over the flow of real power and reactive power over the network has been established.

B. Definitions

Certain terms will be employed repeatedly in the pages to follow, and it will be well to set down precisely the significance to be attached to these terms. For this reason certain definitions will now be given.

Non-Loop Network -- The Non-loop or radial network is a network such that energy may arrive at a given point from a single given source by only one path.

Loop Network -- A loop network is a network which allows energy to be supplied at a given point from a single given source by two or more different paths.

Vector Transformation -- A vector transformation is a transformation which may involve a change in the complex value of the transformation constant.

Phase Transformation -- A phase transformation is a transformation which involves only a change in the argument of the transformation constant.

Magnitude Transformation -- A magnitude transformation is a transformation which involves only a change in the absolute value of the transformation constant.

C. Types of Transformations

The electrical engineer often expresses complex numbers in terms of their vector representations and has become accustomed to speak of complex quantities in terms of the vectors representing these complex quantities. For this reason the terms vector power, vector current, and vector voltage have come to replace the terms complex power, complex current, and complex voltage in engineering usage.

The transformation capabilities of the power transformer, a non-rotating piece of electric machinery, has been the major factor in the preponderant use of alternating current in transmission systems.

A single core power transformer of the type usually used in connection with the transformation requirements of single-phase systems is indicated diagrammatically in Figure 1. Transformers of this type, in their idealized form, are capable of producing voltage and current transformations which are given by the complex transformation equations,

$$E_2 = a\ E_1 \qquad (1)$$

$$I_2 = \frac{1}{a}\ I_1 \qquad (2)$$

where E_1 is the value of the induced e.m.f. in the primary winding of the transformer and I_1 is the value of the primary current. E_2 and I_2 are the corresponding values for the secondary winding. The transformation ratio a which in this case of a single-phase transformer is a scalar number is determined by the ratio of the number of secondary turns to the number of primary turns on the windings of the transformer. The value of the transformation constant is given by the familiar scalar equation:

$$a = n_2/n_1 \qquad (3)$$

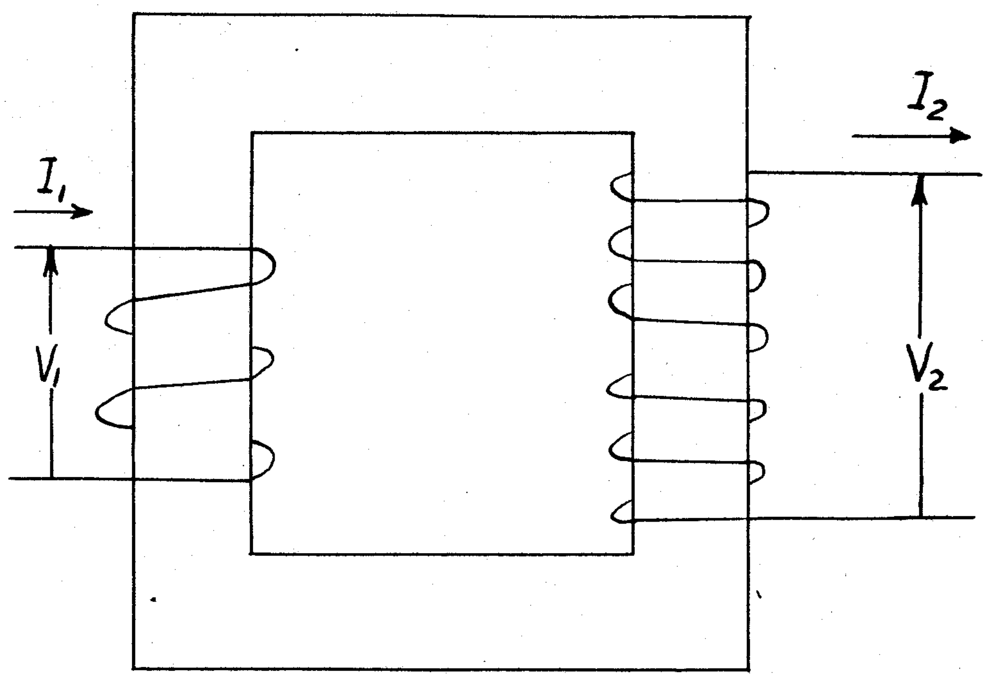


Figure 1. Single Phase Transformer

In the above equation $\underline{n_1}$ is the number of turns wound on the primary winding and $\underline{n_2}$ is the number of turns wound on the secondary winding of the given transformer.

It will be observed that the nature of this transformation is such as to retain the secondary volt-ampere product invariant with respect to the primary volt-ampere product regardless of value of the transformation or the character of operation. As viewed in the complex voltage and complex current planes the transformation results in a simple stretching or shrinking of the vector representing the complex voltage with the corresponding inverse effect occurring in the current plane. Stretching occurs in the complex voltage plane when the transformation constant exceeds unity. Shrinking occurs in the voltage plane for values of the transformation constant less than unity.

There is no relative rotation of the current or voltage vectors involved in such a transformation provided that the transformation be performed by a transformer free of imperfections. A close approach to this condition is possible in practice. The transformation thus produced is represented graphically in the complex plane representations of voltage and current shown in Figures 2-A and 2-B.

Transformers for other than single-phase circuits may be readily designed to produce a more general type of transformation. Such a transformer for use with a three-phase system is illustrated diagrammatically in Figure 3. It will be seen to consist of more than one core, and is in fact a combination of several single-phase

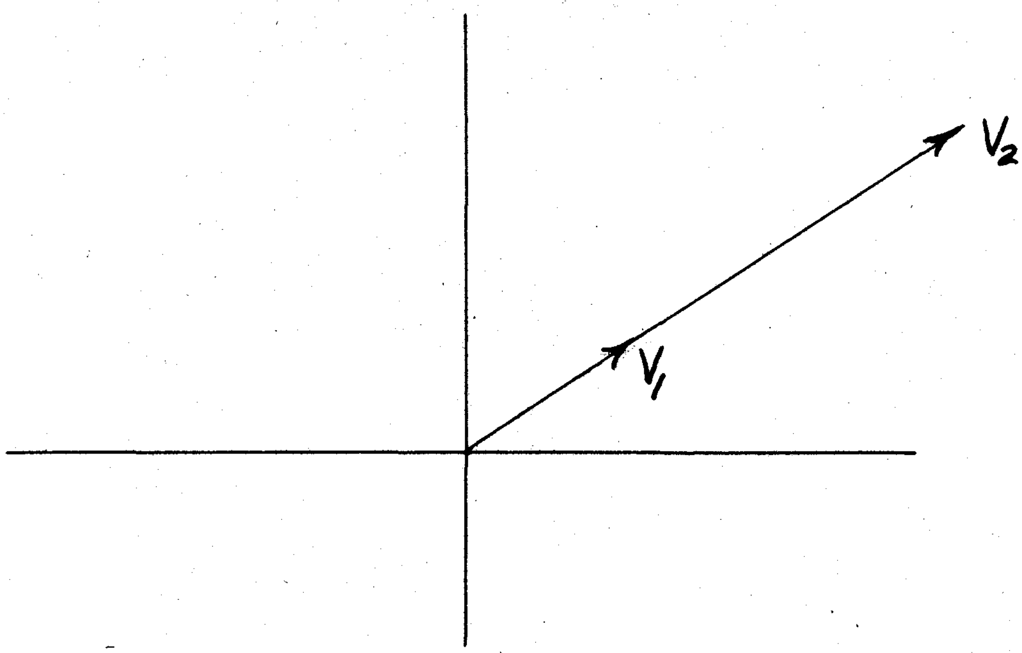


Figure 2-A. Complex Voltage Plane

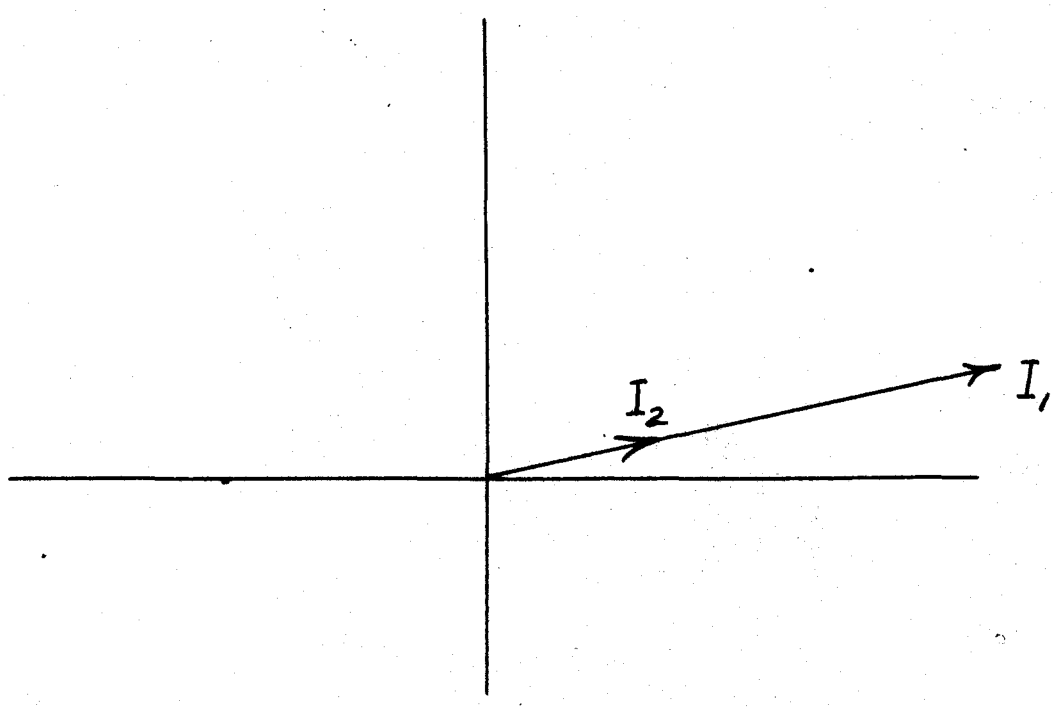


Figure 2-B. Complex Current Plane

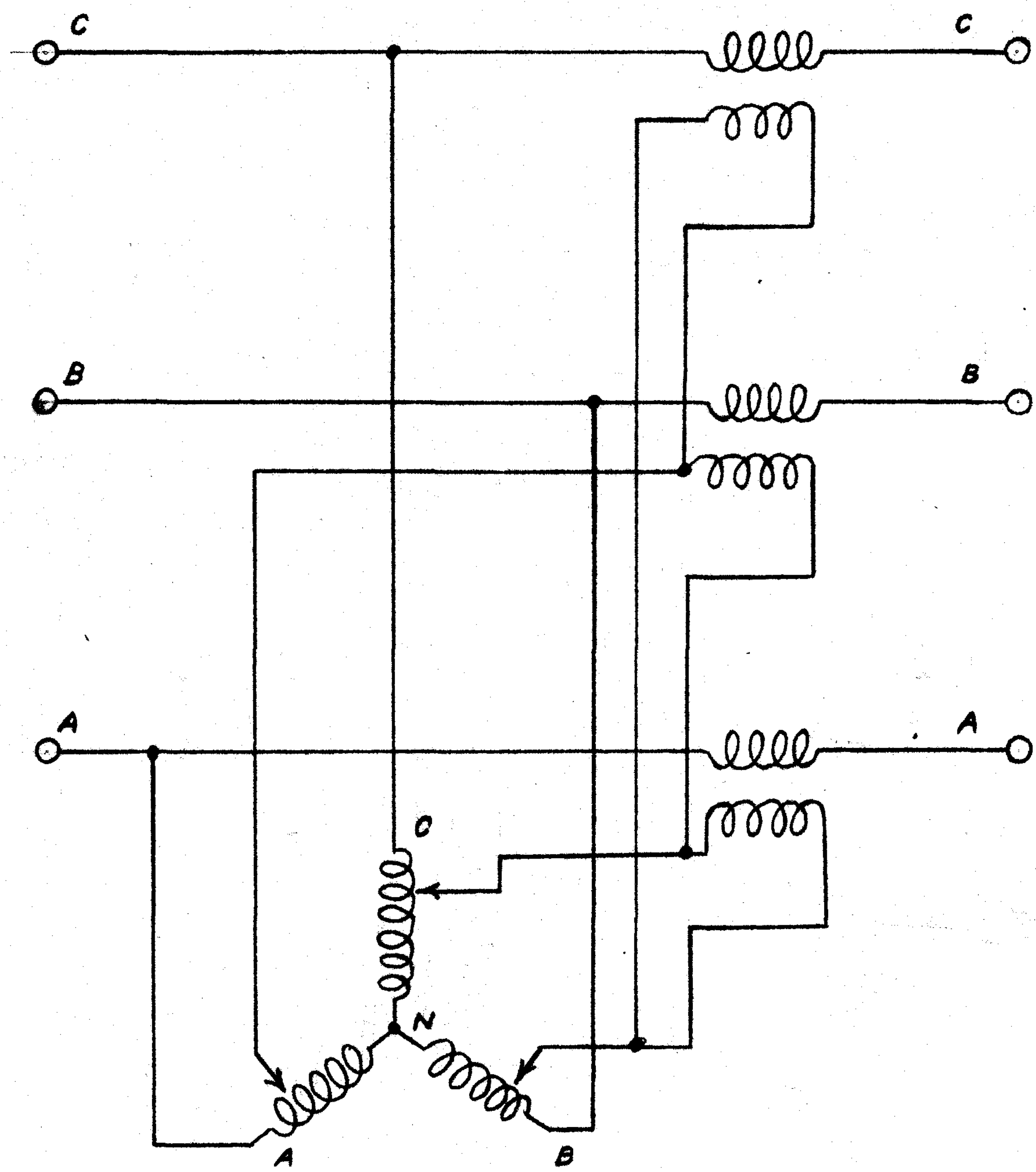


Figure 3. Vector Transformer

transformers.

Transformers of this type, in their idealized form, produce current and voltage transformations which are given on a per-phase basis by the complex transformation equations:

$$E_2 = a\ e^{j\alpha} E_1 \tag{4}$$

$$I_2 = \frac{1}{a}\ e^{j\alpha} I_1 \tag{5}$$

or by the following complex transformation equations:

$$E_2 = A\ E_1 \tag{6}$$

$$I_2 = B\ I_1 \tag{7}$$

where,

$$A = a\ e^{j\alpha} \tag{8}$$

$$B = \frac{1}{a}\ e^{j\alpha} = \frac{1}{\bar{A}}\ e^{j2\alpha} \tag{9}$$

Here the voltage transformation constant $\underline{A}$ must be regarded as a complex number with both real and imaginary parts. Indeed, the voltage transformation constant $\underline{A}$ and the current transformation constant $\underline{B}$ may both be represented on an Argand diagram which we may choose to call the <u>complex transformation plane</u>, or perhaps, in deference to engineering usage, the <u>vector transformation plane</u>. In the complex transformation plane the complex current transformation constant $\underline{B}$ will be found to be the inverse with respect to the unit circle of the voltage transformation constant $\underline{A}$. Upon examination of the $\underline{E_s}$ and $\underline{I_s}$ vectors in their respective planes of their complex plane representation it will be found that they have undergone not only the appropriate stretching or shrinking, but that they have both undergone a rotation by the transformation angle $\underline{a}$ with respect to the primary values. The magnitude of the input and output volt-ampere product again remains invarient, but the phase of the secondary volt-ampere product, a double frequency quantity with respect to voltage and current, has undergone the corresponding angular shift of 2 $\underline{a}$ at this double frequency. A transformation of this type is illustrated graphically by the complex plane representations of Figures 4-A, 4-B, and 4-C.

These transformations which involve the complex transformation constants, $\underline{A}$, and $\underline{B}$, are of a very general type, and we shall term them Vector Transformations.

There are two important sub-classes of vector transformations. The first of these is the vector transformation which does not involve angular rotation, i.e. the angle $\underline{a}$ is equal to zero. These are known as magnitude transformations because only the magnitude of the vectors upon which the transformation operates undergoes alteration. These are the transformations which were first discussed in this section, and which are given by Equations (1) and (2).

The second important sub-class of the vector transformation is the phase transformation. Such transformations do not involve stretching or shrinking of the vectors concerned in the transformation but do involve angular rotation. The phase transformation is expressed

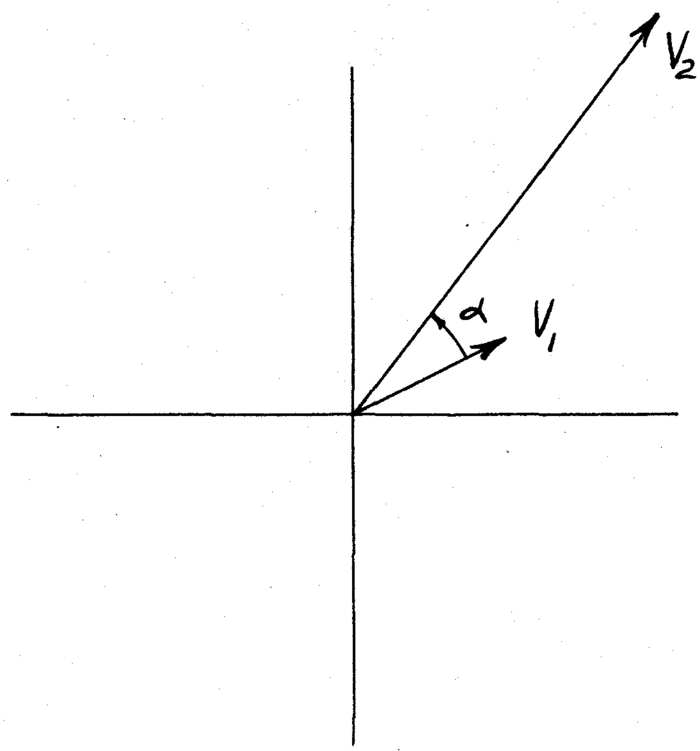


Figure 4-A. Complex Voltage Plane

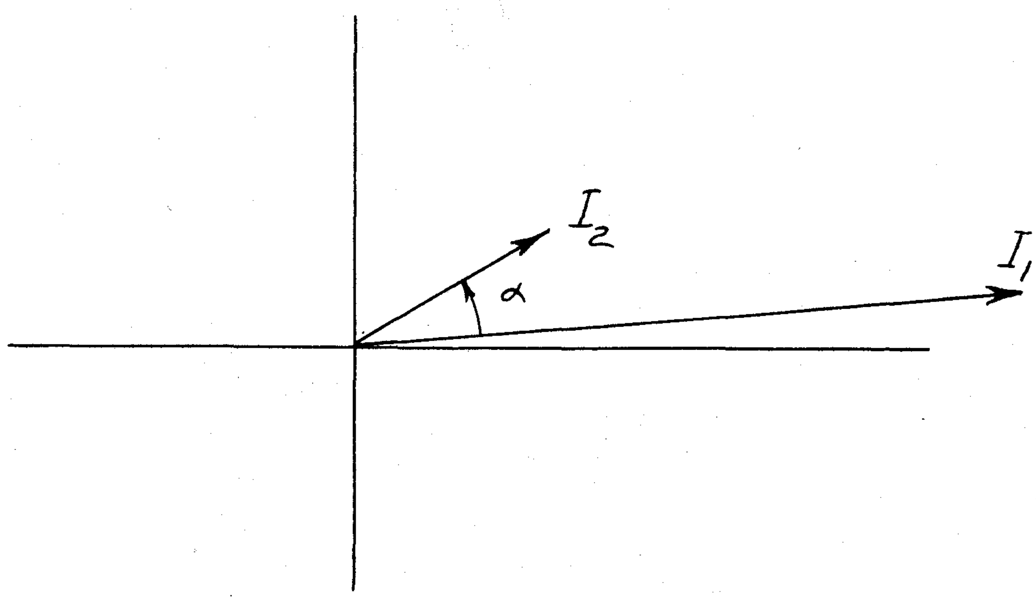


Figure 4-B. Complex Current Plane

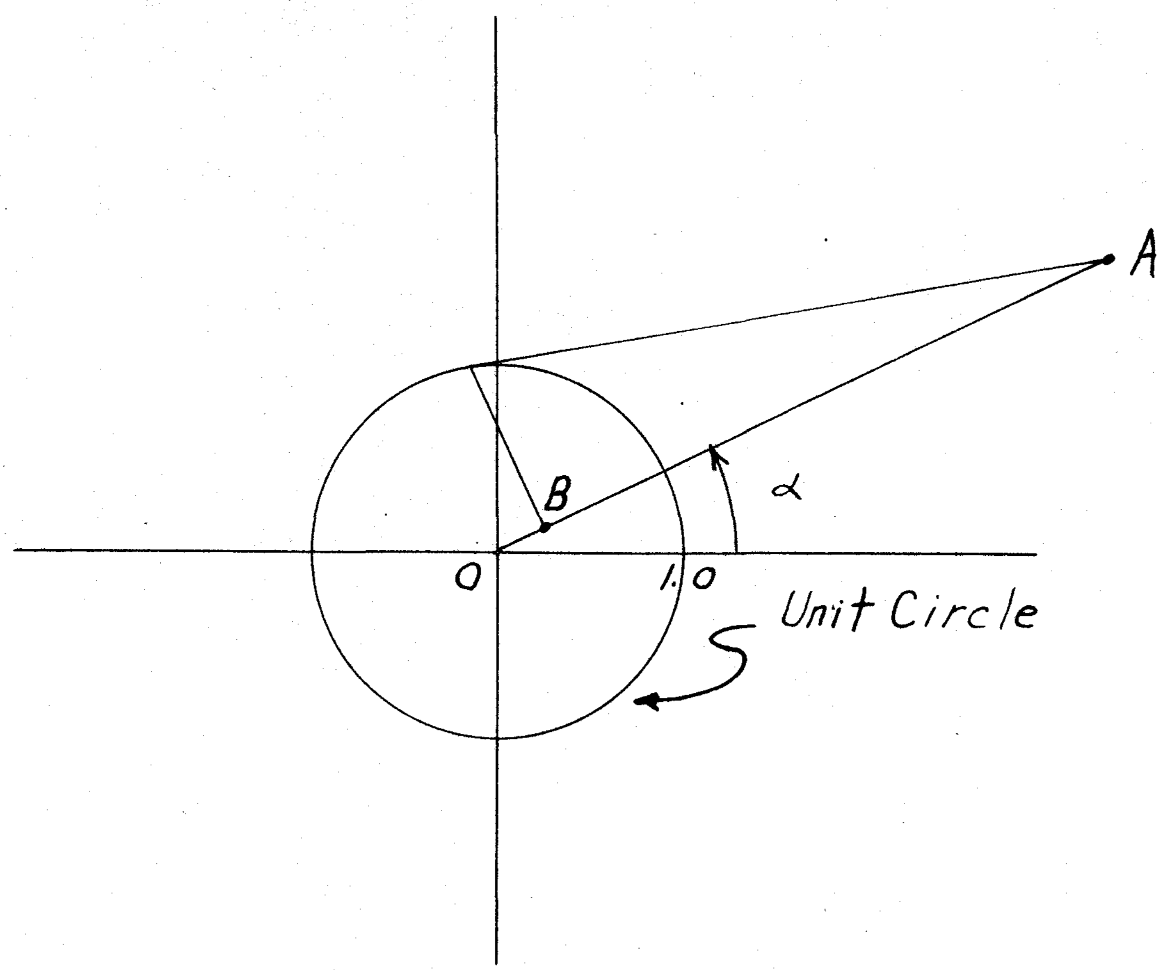


Figure 4-C. Complex Transformation Plane

by the relations:

$$E_2 = e^{j\alpha} E_1 \tag{10}$$

$$I_2 = e^{j\alpha} I_1 \tag{11}$$

A transformation of this type is represented in Figures 5-A, 5-B, and 5-C.

The point marked A in the complex transformation plane of Figure 4-C may be taken to represent graphically the value of the complex voltage transformation constant for some given vector transformation. The angle of the transformation is α, and the magnitude of the voltage transformation constant is given by the length of the vector from the origin to the point A.

The complex current transformation constant B may be easily found by the use of a simple geometrical construction, well known in complex variable theory, for the determination of the inverse with respect to the unit circle.

To find the current transformation constant simply draw two straight lines. Draw the line OA from the origin to the point A and draw a line from the point A such that it will pass tangent to the unit circle. The point on the line OA at which a perpendicular from the line OA passes through the point of tangency found above is the inverse point B. This simple procedure is simply reversed when it is desired to determine a point inverse to an interior point of the unit circle.

It will be observed that both the point A and the point B lie on

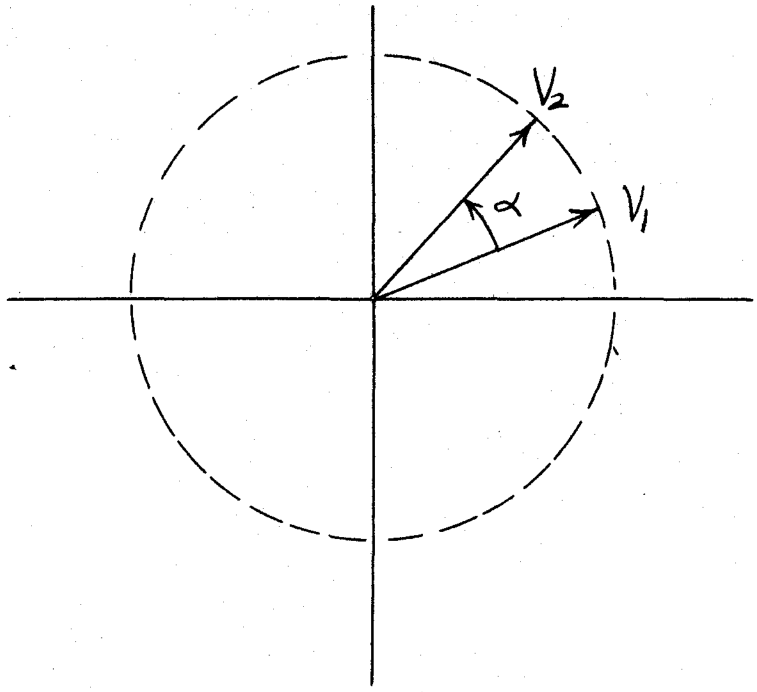


Figure 5-A. Complex Voltage Plane

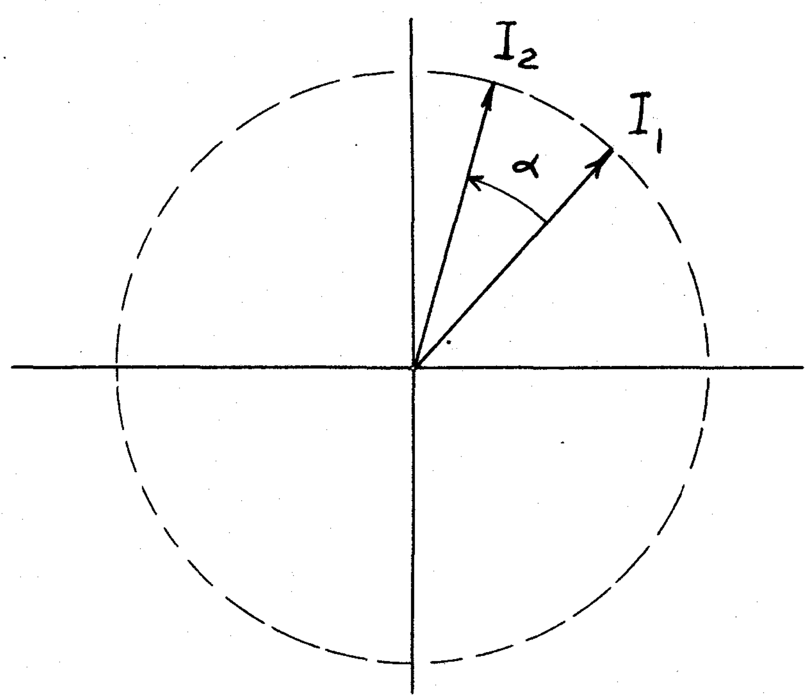


Figure 5-B. Complex Current Plane

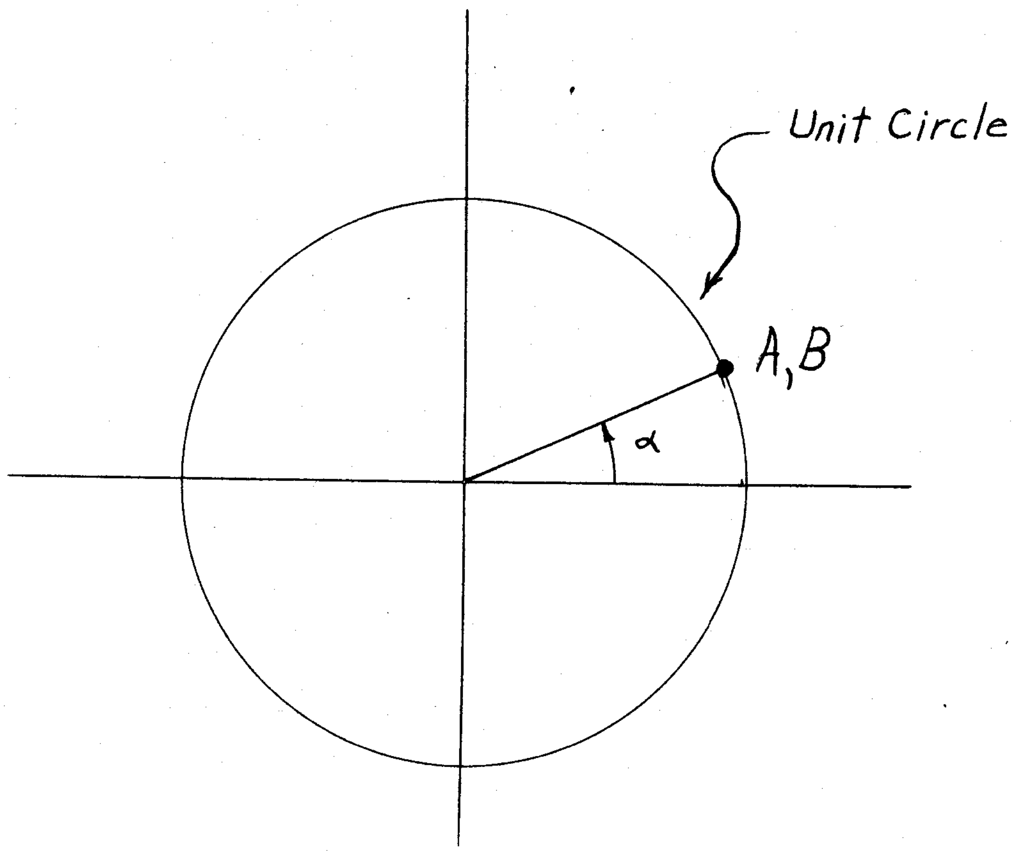


Figure 5-C. Complex Transformation Plane

a common radius vector making an angle _a_, the transformation angle, with the positive axis of reals. Analytically, the product of the distances _OA_ and _OB_ must equal unity.

In the particular case of a pure magnitude transformation the angle _a_ is always zero, and the points _A_ and _B_ lie along the real axis at distances inverse to each other with respect to the point unity.

In the particular case of a pure phase transformation shown in Figure 5-C the points _A_ and _B_ will always coincide with each other and lie on the unit circle at a point such that a radius vector from the origin through _A_ and _B_ makes an angle _a_ with the positive real axis equal to the value of the phase transformation angle.

The point _A_ in the case of a general vector transformation may lie anywhere in the complex plane. The current transformation constant _B_ under a vector transformation will always be the point inverse to _A_ with respect to the unit circle.

A third sub-class of the vector transformation, the _identity transformation_, exists. Its properties are not of such a nature as to be of further interest here. The equations of the identity transformation, a transformation which involves neither magnitude nor angular changes, are:

$$E_2 = E_1 \tag{12}$$

$$I_2 = I_1 \tag{13}$$

Identity transformations are frequently employed to isolate portions of a network for the purposes of grounding. Transformers used for this purpose are usually referred to as isolation transformers.

D. Transformation Effects

Magnitude transformations can immediately be seen to be transformations of the most far-reaching significance. For almost all applications the magnitude of the voltage to be made available is a primary consideration. Any ordinary voltage magnitude requirement can be met by simply providing a transformer of the appropriate turns ratio such that it will supply energy at the required voltage from a source which may have a considerably different voltage. In power transmission and distribution practice it has very often been found wise to construct this transformer in such a way that its ratio may be adjusted to accommodate the alterations that the source voltage may undergo as a result of variable loading.

The distribution of average vector power over the network is not influenced by angle transformations in the case of a single radial system.

The distribution of average vector power, particularly real power, over a network consisting of two radial systems interconnected by a single path will depend upon the relative phases of the e.m.f.'s generated by the generators of these individual radial systems. The angles of these e.m.f.'s are readily controlled by appropriate governor settings of the prime movers to maintain a desired distribution of real power between the generating systems. The contribution of each generator to the reactive power supplied to the network of such a system, as is well known, may be apportioned by raising or lowering the e.m.f. of a given generator either by altering its excitation or by allowing it to supply the network through a transformer of adjustable ratio.

It is however, the loop type circuit which displays the most interesting behavior under the influence of magnitude and angle transformations. The behavior of loop systems is less well known and considerably more difficult to determine than that of the simple radial or the simply interconnected (non-loop) system. It will be the purpose of this thesis to consider loop systems in detail with a view to the determination of their behavior under the influence of vector transformations whose product around every closed loop is not real unity.

E. Loop System Behavior

A simple loop system is shown diagrammatically in Figure 6. Let A-A' and B-B' represent transmission lines from a generator, G, to L_1 and L_2. T_1 is a vector transformer. This simple loop system will serve to illustrate the effect of the vector transformation upon the power flow in systems which involve closed loops.

If the loop is left open at the load end it is possible to adjust the vector transformer T_1 such that the value of e, indicated on the figure as the voltage across switch S, is zero. Under these circumstances the switch S may be closed, and the currents and voltages over the entire system will remain unaltered with the closing of the loop. The conditions under which the system is operating may reasonably be those indicated in Figure 7.

If transformer T_1 is adjusted to give an in-phase increment of

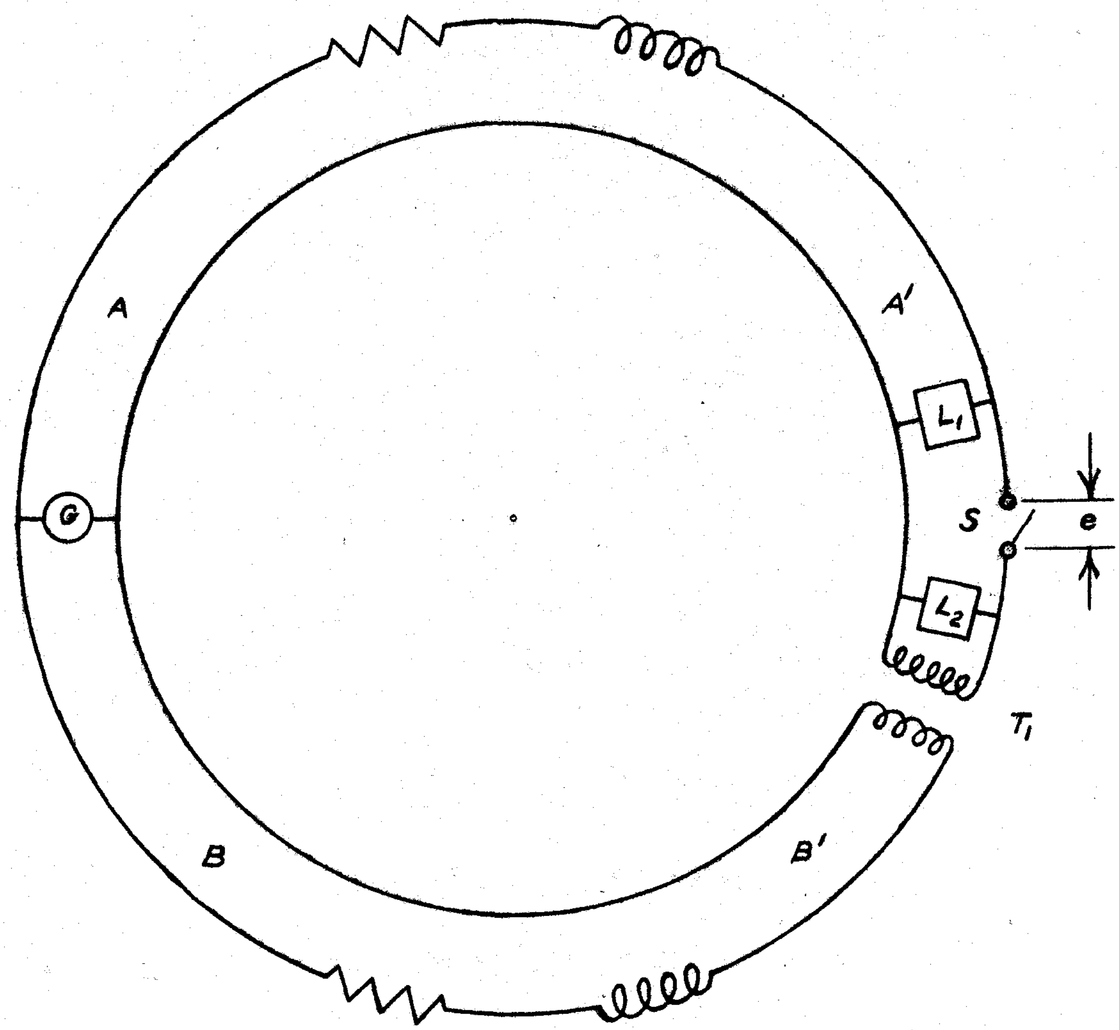


Figure 6. Simple Loop System

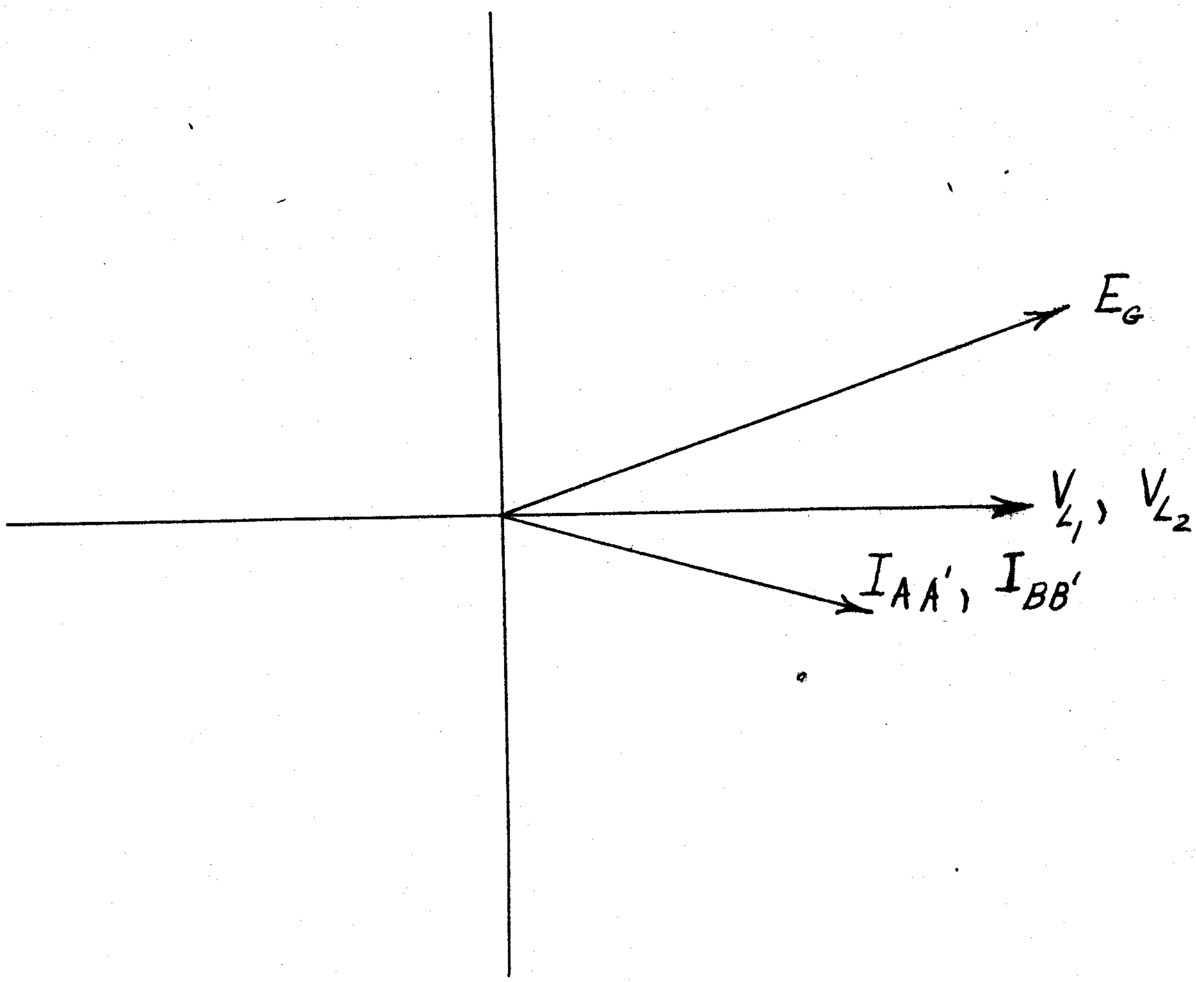


Figure 7. Loop Operation With No Circulating Current

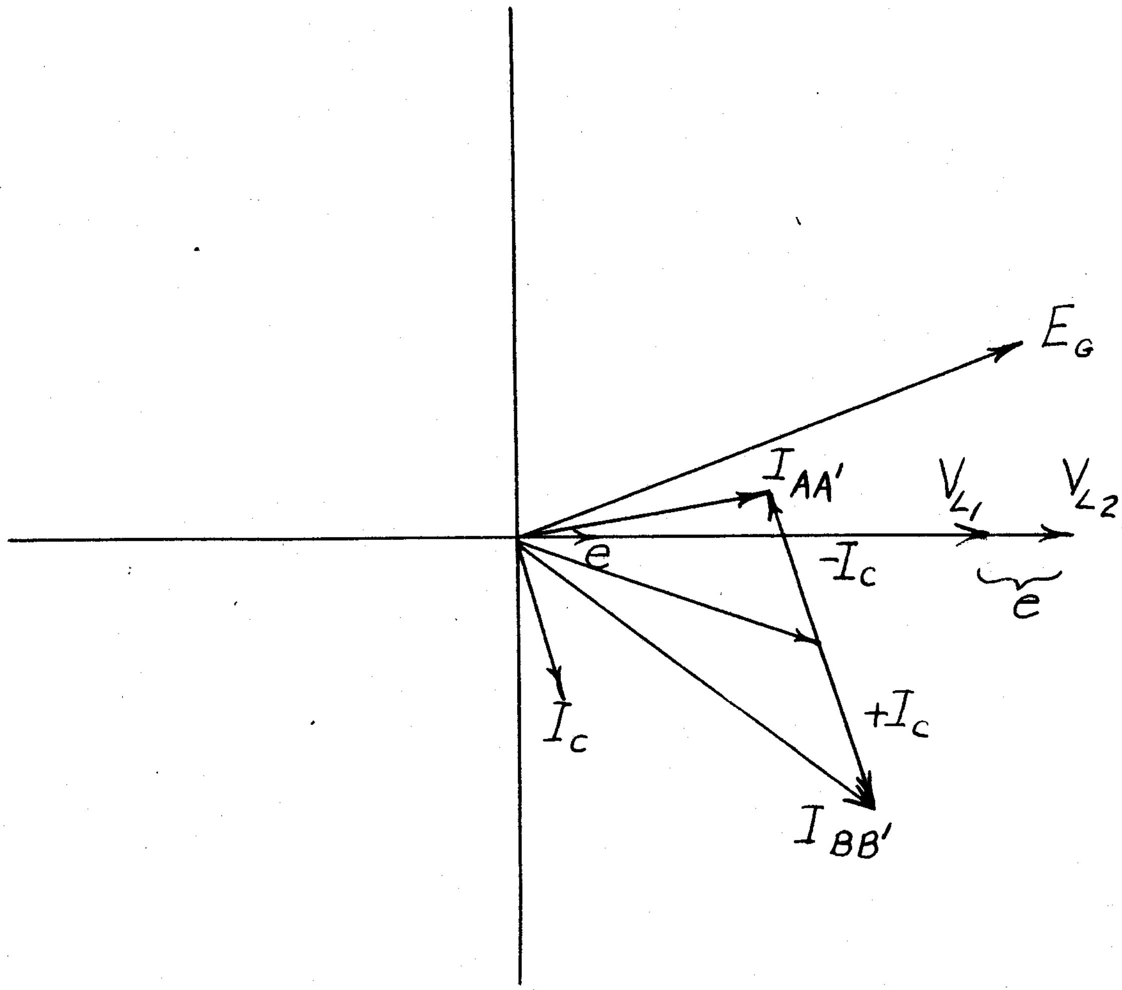


Figure 8. Loop Operation Under Magnitude Transformation

voltage e across the switch prior to the closing of the switch then upon closing the switch this voltage will cause a current to circulate through the system. Most of the current will circulate around the low impedance path formed by the transmission lines. As these transmission lines are normally quite inductive this current, I_c will ordinarily lag the voltage producing it by nearly 90°. Thus this circulating current adds almost directly to and subtracts almost directly from the quadrature components of current already flowing in lines A-A' and B-B' as is shown in Figure 8. The net effect has been an increase in the reactive power supplied by line A-A' and a reduction of reactive power supplied by line B-B'. The important conclusion which has been illustrated is that magnitude changing transformers may be used to apportion the flow of reactive power on loop systems in systems having large X_L/R ratios in their transmission lines.

If instead of allowing T_1 to produce an in-phase increment of voltage e it is adjusted to produce a small quadrature voltage difference of e before closing the switch the behavior is quite different. This situation is shown in Figure 9. Again a circulating current I_c is produced which lags the voltage which has produced it by almost 90° in the predominantly reactive loop circuit. Again, as before, the highly inductive, low impedance, lines present the most important path for this circulating current.

It is evident that in this case the in-phase component of the current in line A-A' is quite decidedly increased, and the in-phase component of the current in line B-B' is correspondingly decreased.

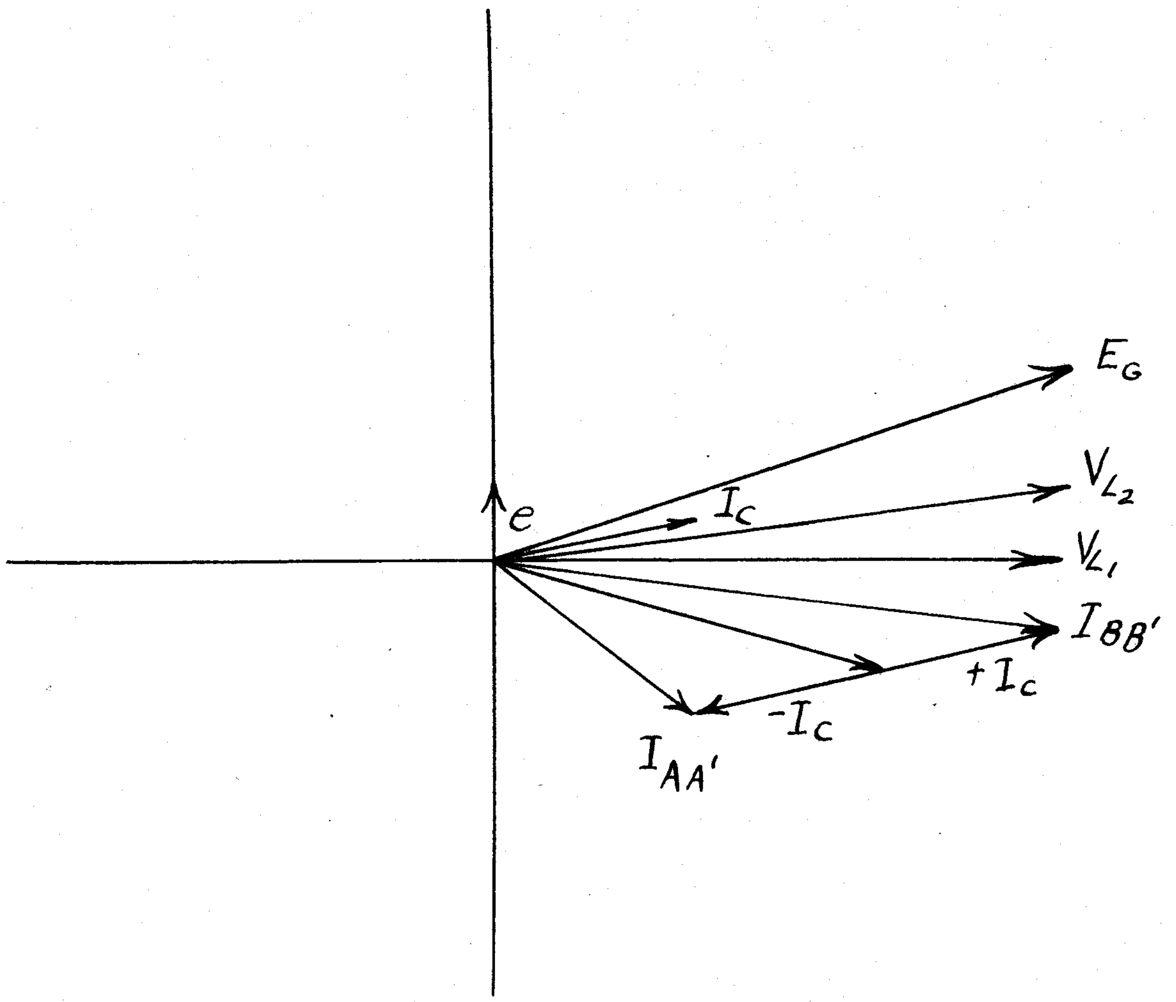


Figure 9. Loop Operation Under Phase Angle Transformation

A second important conclusion can be drawn. It is that phase transformations allow the control of the distribution of real power over a network which has large X_L/R ratios in its transmission lines. The point should be made here that only the distribution of real power flow can be controlled. The total amount of real power flowing over a network depends only on the governor settings of the prime movers of the machines involved.

II. REVIEW OF THE LITERATURE

The interconnection of systems became general in the 1920's. The question of controlling power flow on interconnected systems of the type that close upon themselves was immediately encountered. Blume's[1] pioneer paper in 1927 discussed the advantages gained by employing ratio (magnitude) and phase transformers in various network situations. Blume derived the criteria for minimum system loss and showed that in a resistive loop, or even in a loop in which the ratio of the inductive reactance to the resistance for all portions of the loop are equal, the current distribution for minimum loss occurs naturally. Blume also derived the relation which must be satisfied if voltage is to be injected into a loop to make the system losses a minimum.

The difficulty in designing systems involving phase angle control prompted West[25] to remark in 1930 that the greatest problem was not with the equipment, which he asserted was available in reliable and simple form, but rather with the calculation of the amount of phase shift required to satisfy certain predetermined conditions.

Records were made of power flow in large loop systems and Lyman published a set of data gathered in tests of such a system in 1930. Lyman[16] used his data to calculate an "effective impedance", experimentally determined, of the system loop to quadrature voltage.

Wyman[27], in the same year, proposed the terms open interconnection for the circumstance of only one tie between interconnected systems and closed interconnection for those interconnections which involve two or more ties between the interconnected systems.

The interconnection of systems is an important factor in the economy of operation of electric power networks. Keenan[12] has devoted an entire paper to the economics of interconnected systems.

The operation of a closed-loop system was analyzed by Mollang, a Belgian engineer. Mollang[21] employed the circle-diagram method of transmission-line analysis to determine the operation of the system under various conditions of loading.

The Mexican Chapala-Guanajuato network has been described by Fugill[8]. The phase-shifting transformer employed in that system was implemented with induction regulators in order to provide a continuous adjustment between tap positions throughout its entire range of operation.

Lyman and North[18] described in 1938 the installation of a phase-shifting transformer in the Pittsburgh area to permit the operation of an interconnection the closing of which had caused the circulation of an uncontrollable power flow prior to the installation of the transformer.

Church[2] has set forth methods of determining the general behavior of a loop network, and Roodhouse[22] has shown that the installation of a phase shifter enabled the Nebraska Power Company to use existing lines to supply a heavy load that would otherwise have over-loaded one of the two lines supplying this load.

Ferri and Vaughn[7] in 1943 described the operation of a 138 kv, 104.7-ampere phase shifter used in the Akron-Canton, Ohio, area for the control of power flow on an interconnected loop system. This large phase shifter was constructed to provide a phase shift of 12 degrees in 16 equal steps. The phase shifter was installed for the purpose of maintaining the power flow over the tie lines of the three interconnected companies in accordance with the contractual stipulations among those companies.

The Central Station Engineers of the Westinghouse Corporation in the Westinghouse Transmission and Distribution Reference Book[26] have discussed the question of the solution of networks involving vector transformations. They have proposed a simplified approximate method for the solution of such circuits for the special case of small residual phase angles or small transformation ratios resulting from the complete traversal of the transformations of a given loop.

Hobson and Lewis[10] have devoted some attention to the solution of circuits involving regulating transformers. They have shown that in the case of symmetrical components a phase transformer shifts the positive sequence quantities positively by the transformation angle, the negative sequence quantities negatively by the transformation angle, and the zero sequence quantities undergo no phase shift whatever.

Kimbark, in the discussion of the article on regulating transformers by Hobson and Lewis, has pointed to the inherent difficulty of representing phase transformations on a single phase system. Kimbark has suggested a method which involves the use of a two phase equivalent circuit.

Westinghouse network analyzer engineers in their network analyzer manual present as alternatives two separate methods of representing a phase changing transformer on a network analyzer.

III. SOLUTION OF LOOP CIRCUITS

A. Analytical Methods

Loop circuits which close, but which in closure involve a net transformation ratio of exactly real unity may be solved by the familiar method of reduction to a common base to eliminate the equations introduced at the points of transformation. Once this solution has been obtained the solution of the circuit in terms of actual quantities is readily obtained from the known ratio of base to actual quantities in all parts of the circuit.

Loop circuits which close, but which in closure do not involve a net transformation ratio of exactly real unity may not be solved for an exact solution by the above method as all of the circuit constants may not be expressed in terms of a single common base value. Under these conditions the relative transformation ratios exert their own effect to alter the distribution of power over the system.

Several methods exist for the mathematical solution of loop circuits of which the most fundamental is unquestionably the simultaneous solution of both the circuit and the transformer equations. These solutions, even in the simplest case, will be ponderous as every transformer retained introduces two equations in addition to the circuit equations of the associated network.

Power flow in loop circuits is determinable graphically by the application of methods well known in power circle diagram theory. While these methods are among the more convenient methods of solution, they are less satisfactory because they are graphical in nature.

Loop circuits which involve only a very small incremental transformation are commonly solved by an approximate method. The effect of an external generator upon the system is computed. This external generator supplies an incremental e.m.f. identical in both magnitude and phase with that supplied by the transformer. For comparatively small inserted values of e.m.f. this method will yield satisfactory approximate answers. It becomes increasingly in error, however, as increasing e.m.f.'s are supplied from the external source. These additional watts and vars supplied by the external generator produce errors over the system.

The technique to be set forth in this thesis, the two generator method, is equivalent to the method above except that an additional "generator" has been incorporated that absorbs the volt-amperes introduced by the incremental e.m.f. generator.

B. Analyzer Methods

Loop circuits which involve only magnitude transformations present no problem in their solution by a network analyzer. It is only necessary to employ a transformer or an auto-transformer of appropriate ratio to represent the transformation conditions which are in existence in the actual circuit.

In the case of loop circuits which involve angular transformations the problem is less easily solved. Quadrature components of e.m.f. are not so readily available in a single-phase analyzer as they are on a three-phase system. This is the basic reason why such transformations are more difficult to represent.

Several methods are in common use to effect a representation of the phase-shifting transformer in analyzer studies. One of the most common of these methods utilizes a load and a generator in cascade to represent the transformation. The load and generator are both adjusted until the terminal conditions at the load advanced by the phase transformation angle are identical with the terminal conditions at the generator. Such a scheme requires the adjustment of four variables in order to reach a proper agreement of their values across the boundary represented by the transformer. Not only are these adjustments mutually interdependent, but their adjustment will also affect other adjustments throughout the system.

The problem of manual adjustment using such a scheme clearly becomes a ponderous one.

Occasionally problems may be solved by analyzer methods neglecting the effect of phase transformations. The phase transformations are then taken into account by analytic means.

There is a second generally used method for the representation of phase transformers. In this method a series generator is employed. The e.m.f. generated by this generator is in quadrature with the input e.m.f. The input circuit is tuned to resonance by means of a parallel reactance. The input current is then in phase with the input voltage. The external series generator then generates an e.m.f. in quadrature with the current through the generator. Under these circumstances only quadrature power is inserted by the external generator. This quadrature power and that taken by the input tuning reactance are then corrected by a second reactance across the output circuit. The result is a shift in phase of current and voltage with no net change in either real or quadrature power. Again, however, the adjustment of four interdependent variables is required in order to secure the proper conditions of operation for any given phase shift angle.

In this work the writer will present a two-generator equivalent circuit for representing phase transformations. This method is particularly adaptable to those high frequency analyzers which employ vacuum-tube generators and vacuum-tube circuits.

IV. THE TWO-GENERATOR EQUIVALENT OF THE GENERAL VECTOR TRANSFORMER

A transformer capable of producing conditions at its terminals which satisfy exactly the transformation equations is called a "perfect transformer", and it is usually represented by the symbol for two coupled coils. Because of inescapable imperfections such as winding resistance, flux leakage, core reluctance and core loss the behavior of an actual transformer is never that of a perfect transformer although it may approach it quite closely. These imperfections produce the same effects that would be produced by impedances in series and in parallel with the terminals of a perfect transformer in so far as the terminal behavior is concerned. In fact an equivalent network may be devised by the proper combination of a perfect transformer with series and parallel impedances which will display the same behavior at its terminals as the actual transformer that it is designed to represent. Such an equivalent circuit is shown in Figure 10-A. The parallel impedance shown is of high absolute value in the case of a properly designed transformer, and its effect is normally of little significance in evaluating the behavior of the transformer under conditions other than zero or very light load. Thus the equivalent circuit for a loaded transformer may be represented by the two series impedances and a perfect transformer as shown in Fig. 10-B.

In this analysis it will not be necessary to consider the effect

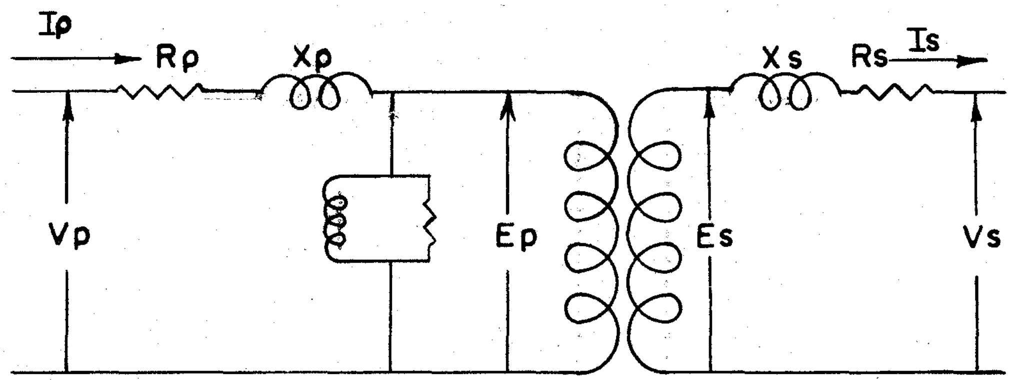


Figure 10-A. Equivalent Circuit of Transformer

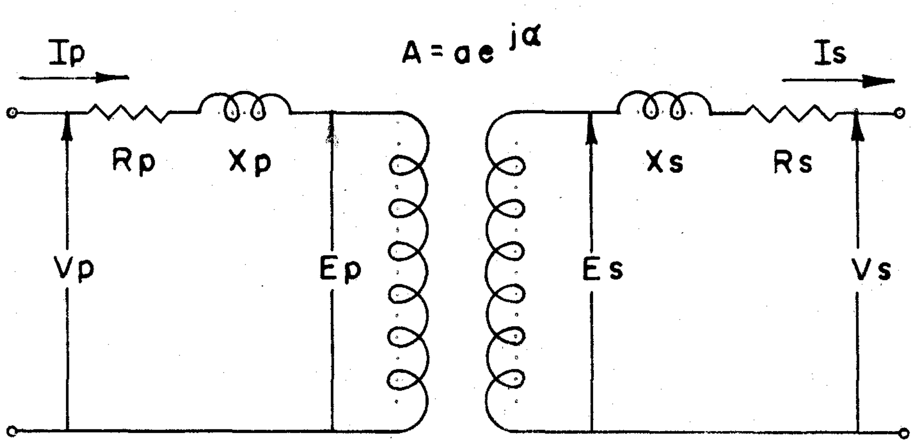


Figure 10-B. Equivalent Circuit of Loaded Transformer

of the equivalent series impedances of the primary and secondary of the transformer. It will be necessary only to work with that portion of the representation which is the perfect transformer. The equivalent series impedances may be conveniently added to the external network when the analysis involves an actual transformer, as may the shunt branch.

Consider the perfect transformer of Figure 11. The primary voltage and current may be given as $\underline{E_p}$ and $\underline{I_p}$. The corresponding secondary values of voltage and current as $\underline{E_s}$ and $\underline{I_s}$. The secondary values may be expressed as the vector sum of the corresponding primary value and some vector fraction of the primary value. Thus,

$$E_s = E_p + \triangle E_p \tag{14}$$

and

$$I_s = I_p + \delta I_p. \tag{15}$$

By definition, under a vector transformation,

$$E_s = ae^{j\alpha}E_p \tag{16}$$

and

$$I_s = \frac{1}{a}\, e^{j\alpha}I_p. \tag{17}$$

This readily allows the development of the basic volt-ampere requirement of the vector transformer as,

$$E_s I_s = E_s I_s \tag{18}$$

$$E_s I_s = (ae^{j\alpha}E_p)(\frac{1}{a}\, e^{j\alpha}I_p) \tag{19}$$

$$E_s I_s = e^{j2\alpha}E_p I_p. \tag{20}$$

By the use of the equations first proposed we may express the above equation as,

$$(E_p + \triangle E_p)(I_p + \delta I_p) = e^{j2\alpha}E_pI_p \tag{21}$$

$$E_p(1 + \triangle)I_p(1 + \delta) = e^{j2\alpha}E_pI_p \tag{22}$$

$$(1 + \triangle)(1 + \delta) = e^{j2\alpha} \tag{23}$$

$$(1 + \delta + \triangle + \triangle\delta) = e^{j2\alpha}$$

$$\delta = -(1 - \frac{e^{j2\alpha}}{1 + \triangle}) \tag{24}$$

$$\delta = -(\frac{1 + \triangle - e^{j2\alpha}}{1 + \triangle}). \tag{25}$$

Since, as has been postulated,

$$E_s = E_p + \triangle E_p \tag{14}$$

$$= (1 + \triangle)E_p. \tag{26}$$

The perfect transformer requires,

$$E_s = ae^{j\alpha}E_p = A\ E_p. \tag{27}$$

It is apparent that,

$$1 + \triangle = ae^{j\alpha} = A. \tag{28}$$

Thus, the expression for δ may be written,

$$\delta = -(1 - \frac{e^{j2\alpha}}{A}) \tag{29}$$

$$= -(1 - \frac{1}{a}e^{j\alpha})$$

$$= -(1 - B). \tag{30}$$

The original equations may now be written as,

$$E_s = E_p + \triangle E_p \tag{14}$$

$$I_s = I_p - (1 - \frac{e^{j2\alpha}}{A})I_p. \tag{31}$$

This expression for the secondary voltage and current in terms of the primary voltage and current describes the circuit shown in Figure 12. The circuit of Figure 12 is therefore an equivalent circuit of the perfect vector transformer. It is composed of two generators. One generator is a voltage generator. It supplies the voltage $\underline{\triangle E_p}$ to the circuit. The other generator is a current generator. It supplies the current, $\underline{(1 - \frac{e^{j2\alpha}}{A})I_p}$, to the circuit. The positive directions for each generator are as indicated on the diagram. If a perfect transformer is replaced by this equivalent network the terminal conditions will remain invariant under the substitution.

Regulating or magnitude transformers are more widely used than vector transformers. For this reason the reduction of the equations to that specific case will now be shown. Let us choose to represent by primed values the constants involved in a magnitude transformation as contrasted with unprimed values in the vector case.

For a vector transformation it has been proven,

$$\delta = -(1 - \frac{e^{j2\alpha}}{1 + \triangle}). \tag{24}$$

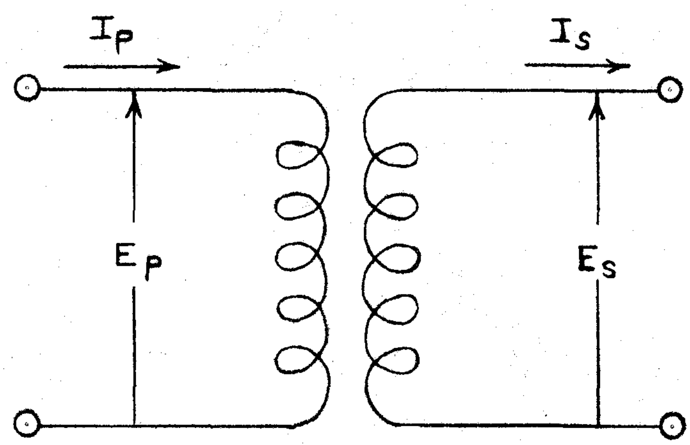


Figure 11. The Perfect Transformer

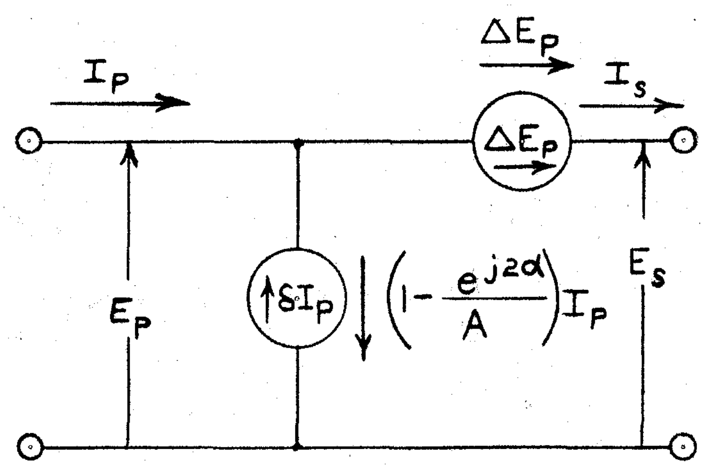


Figure 12. The Two-Generator Equivalent of the Perfect Vector Transformer

Under a scalar transformation,

$$\delta = \delta'$$

$$\alpha = 0$$

$$\triangle = \triangle'.$$

Then.

$$\delta' = -\left(1 - \frac{1}{1 + \triangle'}\right)$$

$$= -\left(\frac{\triangle'}{1 + \triangle'}\right). \qquad (31)$$

The equivalent circuit for a magnitude transformation is therefore that shown in Figure 13. The equations of the magnitude transformation may be written as,

$$E_s = E_p + \triangle' E_p \qquad (32)$$

and

$$I_s = I_p - \left(\frac{\triangle'}{1 + \triangle'}\right) I_p. \qquad (33)$$

The method to be presented for the solution by a network analyzer of problems involving vector transformations requires the resolution of the given vector transformation into its two component transformations -- one a pure phase transformation, the other a pure magnitude transformation. Accordingly, the equivalent circuit of a phase transformer is important here, and it will now be derived. The constants $\underline{\triangle''}$ and $\underline{\delta''}$ which are involved in a pure phase transformation are denoted as double primed quantities in order to maintain distinctly the character of the transformation for which the final results apply.

For the general vector transformation,

$$\delta = -\left(1 - \frac{e^{j2\alpha}}{A}\right) \qquad (24)$$

$$= -1 + \frac{e^{j\alpha}}{a} .$$

By definition for the general vector transformation,

$$1 + \triangle = ae^{j\alpha} \qquad (34)$$

$$\triangle = ae^{j\alpha} - 1. \qquad (35)$$

For a phase transformation a = 1 and so

$$\triangle'' = e^{j\alpha} - 1$$

$$= -1 + e^{j\alpha}. \qquad (36)$$

Thus for a phase transformation,

$$\delta = \delta'' = -1 + e^{j\alpha} = \triangle'' \qquad (37)$$

$$\delta'' = \triangle''. \qquad (38)$$

The equations which express the behavior of a phase transformer become,

$$E_s = E_p + \triangle'' E_p \qquad (39)$$

$$I_s = I_s + \triangle'' I_p. \qquad (40)$$

The equivalent circuit for the case of a pure phase transformation is that shown in Figure 14. In this circuit the positive direction of the current $\underline{\triangle'' I_p}$ has been chosen to retain $\underline{\triangle'' I_p}$ as a positive

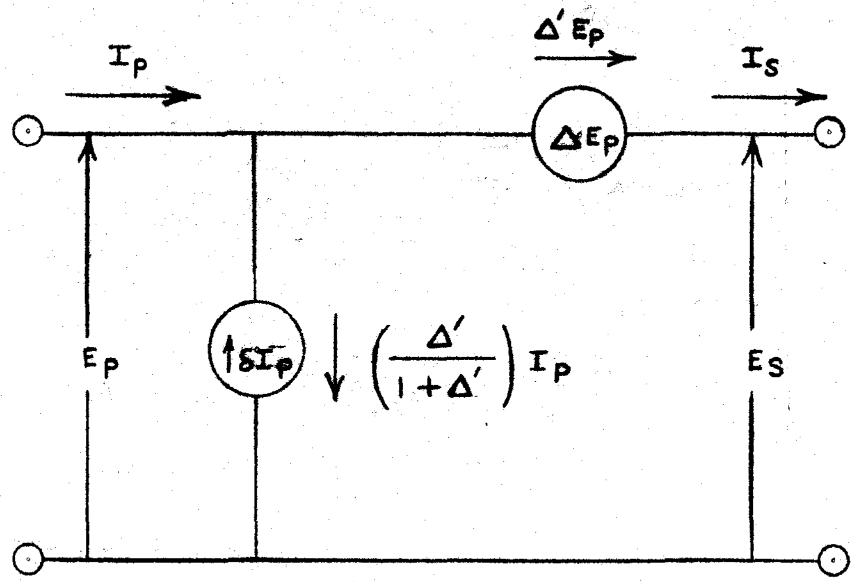


Figure 13. Two-Generator Equivalent of the Magnitude Transformation

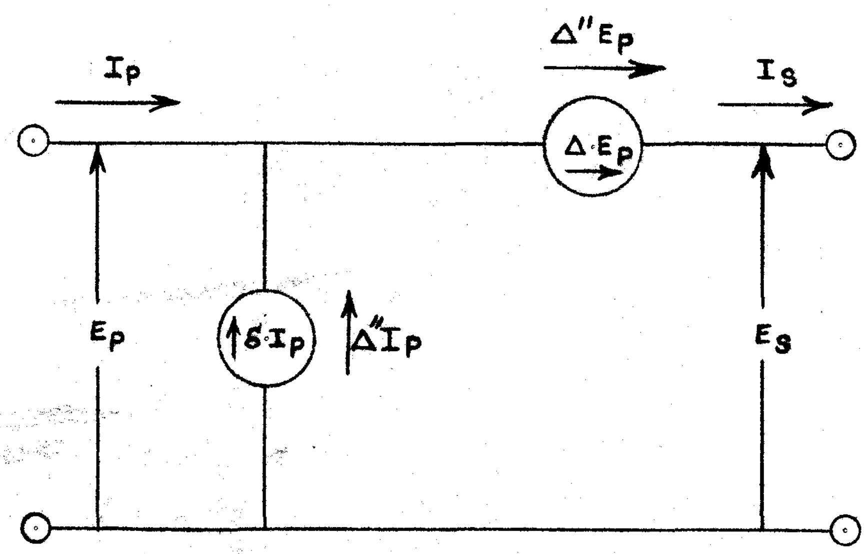


Figure 14. Two-Generator Equivalent of the Phase Transformation

quantity just as the direction was chosen to maintain the corresponding quantities of the previous two cases positive.

The great simplicity of the relations in the particular case of the phase transformation provokes curiosity concerning them. This simplicity among the variables for the case of a phase transformation contributes greatly to the ease with which these transformations, and indirectly vector transformations, may be represented physically on the network analyzer. Further details of this transformation will be presented when the topic of physical representation is considered. For the moment it will be sufficient to consider the problem of determining the general conditions under which the relation $\underline{\delta = \triangle}$ holds.

It has been shown that for the vector transformation,

$$\delta = \frac{-(1 + \triangle - e^{j2\alpha})}{1 + \triangle} \qquad (25)$$

If the condition is now imposed that,

$$\delta = \triangle = \Gamma.$$

We have,

$$\Gamma = \frac{-(1 + \Gamma - e^{j2\alpha})}{1 + \Gamma} \qquad (41)$$

$$\Gamma^2 + 2\Gamma + (1 - e^{j2\alpha}) = 0$$

$$\Gamma = -1 \pm \sqrt{e^{j2\alpha}}$$

$$\Gamma + 1 = \pm\sqrt{e^{j2\alpha}} = \pm e^{j\alpha}. \qquad (42)$$

But for the general vector transformation,

$$1 + \triangle = ae^{j\alpha}. \qquad (28)$$

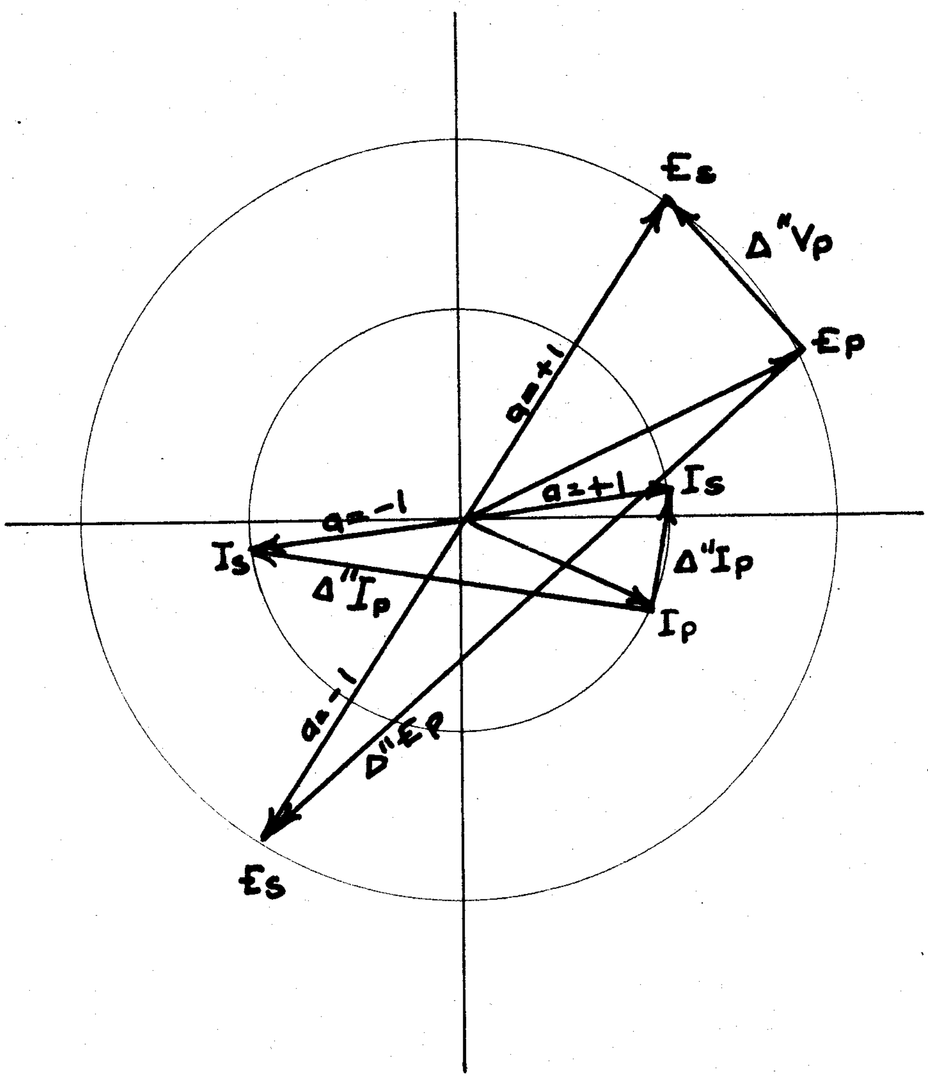


Figure 15. Conditions for the Equality, $\delta'' = \Delta''$

For $\underline{\Delta}$ equal to $\underline{\Gamma}$ this becomes,

$$1 + \Gamma = ae^{j\alpha}. \tag{29}$$

But there has already been derived,

$$\Gamma + 1 = \pm e^{j\alpha}. \tag{42}$$

Since

$$\Gamma + 1 = 1 + \Gamma.$$

So

$$\pm e^{j\alpha} = ae^{j\alpha}$$

$$a = +1, \text{ and } a = -1. \tag{43}$$

It has then been shown that there are two separate conditions of angular transformation for which the per-unit value of the difference between primary and secondary voltage and current have a common value. These situations are illustrated in the vector diagram of Figure 15.

The information which has been presented has given the tools for a mode of attack for the exact solution of those problems which involve loop circuits whose transformation ratios do not form the product of real unity on traversing the loop.

Beyond this, the method allows the use of considerable judgment in simplifying networks for approximate solutions with the consequent vast reduction in computational effort.

A. The Two-Generator Equivalent as an Analytical Method.

It has been shown that any perfect vector transformer may be represented by an equivalent network consisting of two generators.

This equivalent network is shown again in Figure 16. The input voltage, $\underline{E_p}$, may be considered to arise from the contribution of two separate effects of which the voltage $\underline{E_{po}}$ arises from the action of all the sources external to the equivalent network of the transformer, and the voltage $\underline{e_p}$ arises from the joint action of the current and voltage generators of the two-generator equivalent circuit. Thus,

$$E_p = E_{po} + e_p \tag{44}$$

Similarly, the current $\underline{I_p}$ may be considered to arise as the sum of the current produced by the action of all the external generators $\underline{I_{po}}$ plus the current $\underline{i_p}$ arising from the joint action of the two generators of the equivalent circuit. Thus,

$$I_p = I_{po} + i_p \tag{45}$$

The equivalent circuit may then be drawn and labeled as in Figure 17.

The procedure to be followed in determining the performance of a circuit involving a vector transformation may now be outlined. The circuit is first solved under the assumption that $\underline{\triangle} = 0$. From the resulting solution the values of $\underline{E_{po}}$ and $\underline{I_{po}}$ may be immediately determined as well as all of the other currents and voltages in the circuit for this base condition of a real unity transformation ratio. This solution may be termed the "base-ratio" solution.

The circuit is now solved with all of the external generators short circuited, but with the generators of the equivalent circuit

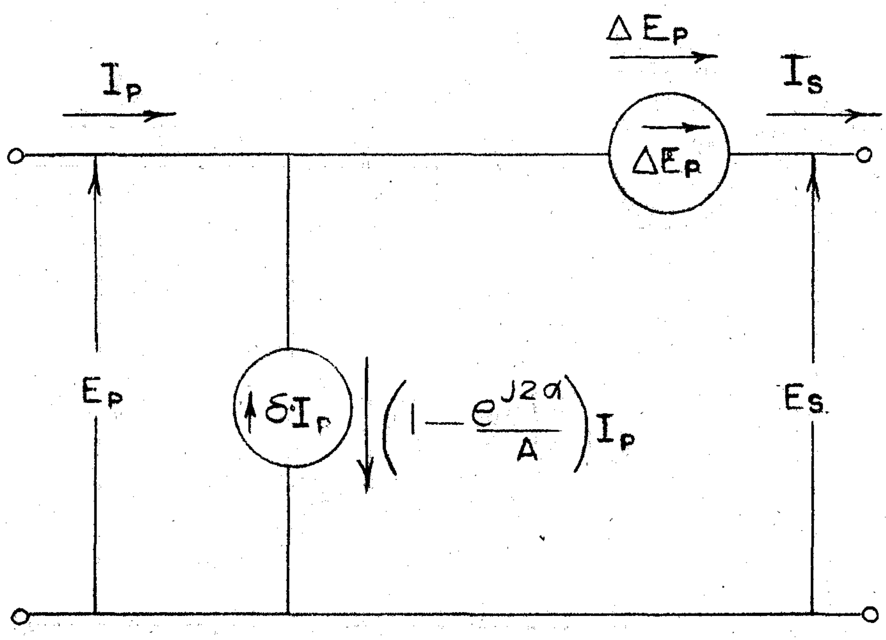


Figure 16. The Two-Generator Equivalent of the Vector Transformer Without Losses

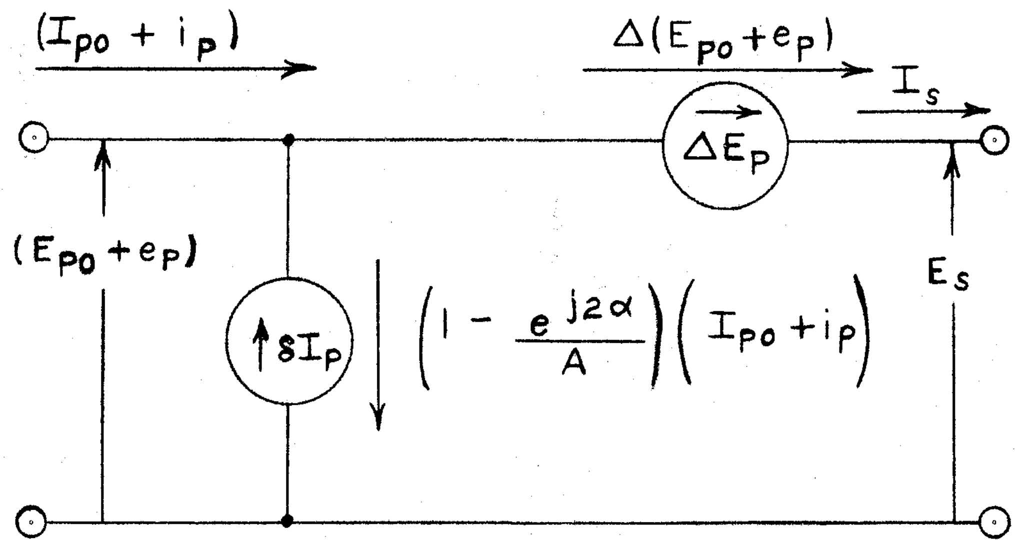


Figure 17. The Two-Generator Equivalent for Analytical Studies

delivering energy to the system. A new set of currents and voltages for the entire system is determined by this solution. This solution is termed the "incremental-ratio" solution.

The exact solution to the problem for any vector transformation ratio is then obtained by superimposing the sets of values obtained for the base-ratio solution and the incremental-ratio solution.

The two-generator equivalent represents only the perfect transformation portion of the equivalent circuit of an actual transformer. Accordingly, the equivalent series impedance and shunt admittance of the equivalent circuit of an actual transformer must not be overlooked as these parameters must be included in both the base-ratio and the incremental-ratio circuit solutions.

A complete solution of a problem involving a vector transformation will in general consist of two component solutions. The first of these two solutions is the base-ratio solution which assumes that the net transformation in traversing the loop under consideration is equal to real unity. Even in the event that the problem is to be solved for several different values of vector transformation this solution need be made only once for any given network.

The second component solution is the incremental-ratio solution, and it is superimposed upon the base-ratio solution to determine the complete solution of the problem. It is evident that a new incremental-ratio solution must be determined each time a new vector transformation ratio is investigated.

B. The Two-Generator Equivalent as an Analyzer Representation of the Phase Transformer

The problem of simulating a vector transformation on a network analyzer has been a difficult one. A vector transformation may be resolved into two component transformations, the one a phase transformation involving pure rotation and the other a magnitude transformation involving pure radial stretching or compression about the origin in the complex transformation plane.

The network analyzer is ordinarily operated as a single-phase device. Under these circumstances the effect of a magnitude transformation is easily represented by the introduction of a magnitude transformer or auto-transformer of appropriate ratio into the circuit. The effect of a phase transformation is not so easy to duplicate since the quadrature components of voltage which are available in the n-phase system are not so available in the single-phase system.

It has been shown earlier, however, that a vector transformer may be represented by an equivalent network involving two related generators. It is from these sources that the required quadrature components may be drawn.

The equivalent circuit which has just been studied is readily adaptable to construction in physical form for use with certain types of network analyzers. The equivalent circuit which may be used is shown in Figure 18. The effect of pure phase transformation which this circuit can produce on the vectors used to represent graphically

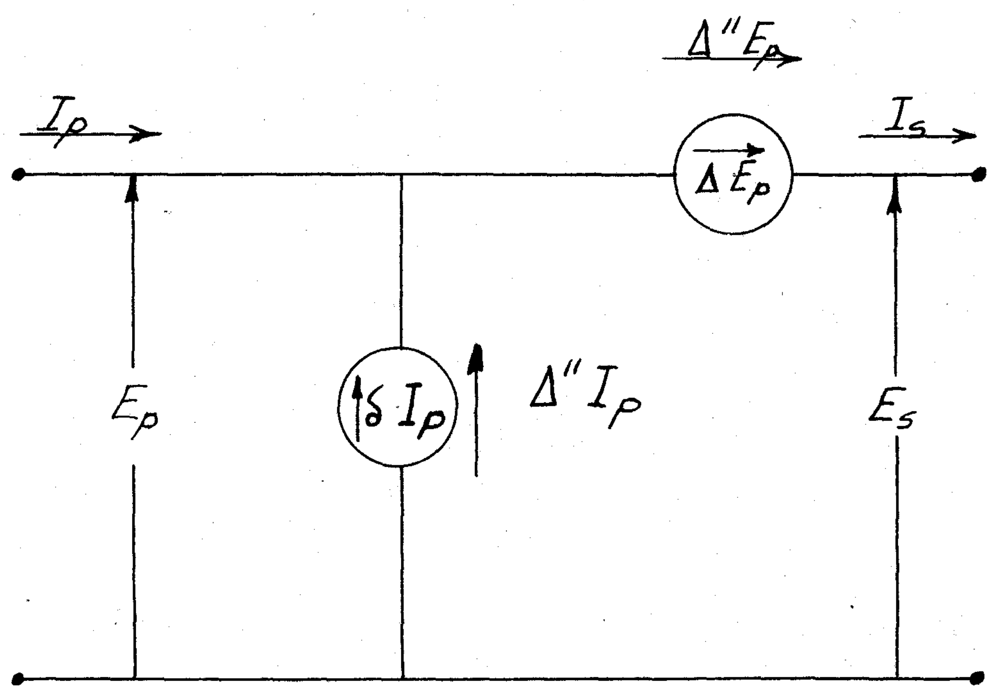


Figure 18. Two-Generator Circuit for Producing Phase Angle Transformations

the complex quantities involved in the transformation is shown in Figure 19.

In order to produce a desired phase transformation angle, α, it is necessary to determine the magnitude and phase angle of the quantity $\underline{\Delta''}$. The quantity $\underline{\Delta''}$ determines the complex values of voltage and current which must be injected into the circuit in the manner shown in Figure 19 in order to achieve the desired phase transformation.

The quantity $\underline{\Delta''}$ is a complex number. As such it has both magnitude and direction. The quantity $\underline{\Delta''}$ may therefore be represented as $\underline{|\Delta''|\, e^{j\beta}}$ where $\underline{|\Delta''|}$ is the absolute value, and $\underline{\beta}$ the phase angle of the quantity $\underline{\Delta''}$ referred to the corresponding E_p or I_p.

The absolute value of $\underline{\Delta''}$ is evident from the simple trigonometry of Figure 20 as detailed below.

$$\operatorname{Sin} \frac{\alpha}{2} = \frac{\frac{|\Delta''\, E_p|}{2}}{|E_p|} \tag{46}$$

$$|\Delta''| = 2 \operatorname{Sin} \frac{\alpha}{2} \tag{47}$$

For small angles,

$$\alpha \approx \operatorname{Sin} \alpha \tag{48}$$

Then approximately,

$$|\Delta''| \approx \alpha \tag{49}$$

Thus the magnitude of $\underline{\Delta''}$ is shown to be a simple function of the phase transformation constant alone. It will next be necessary

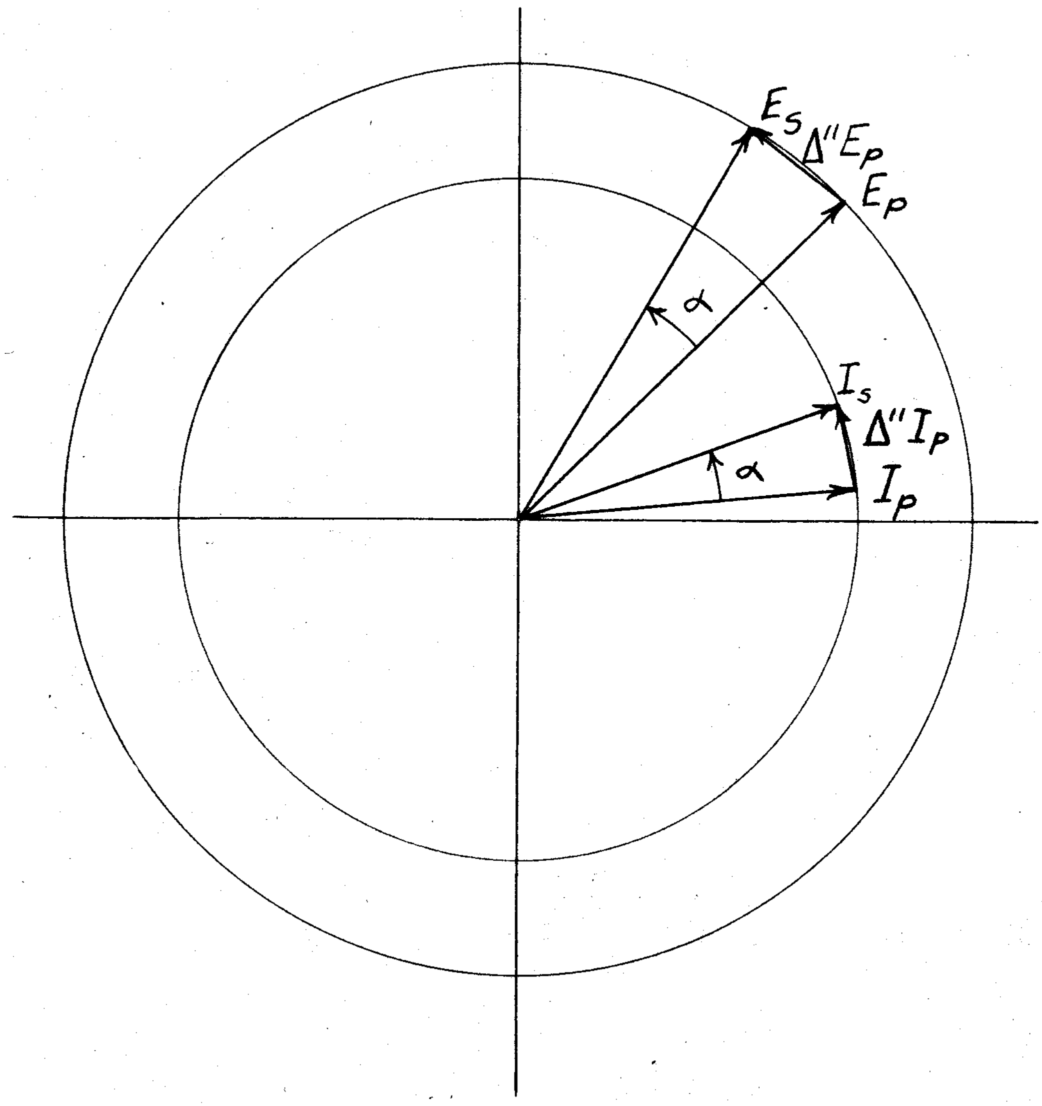


Figure 19. Phase Transformation by the Two-Generator Method

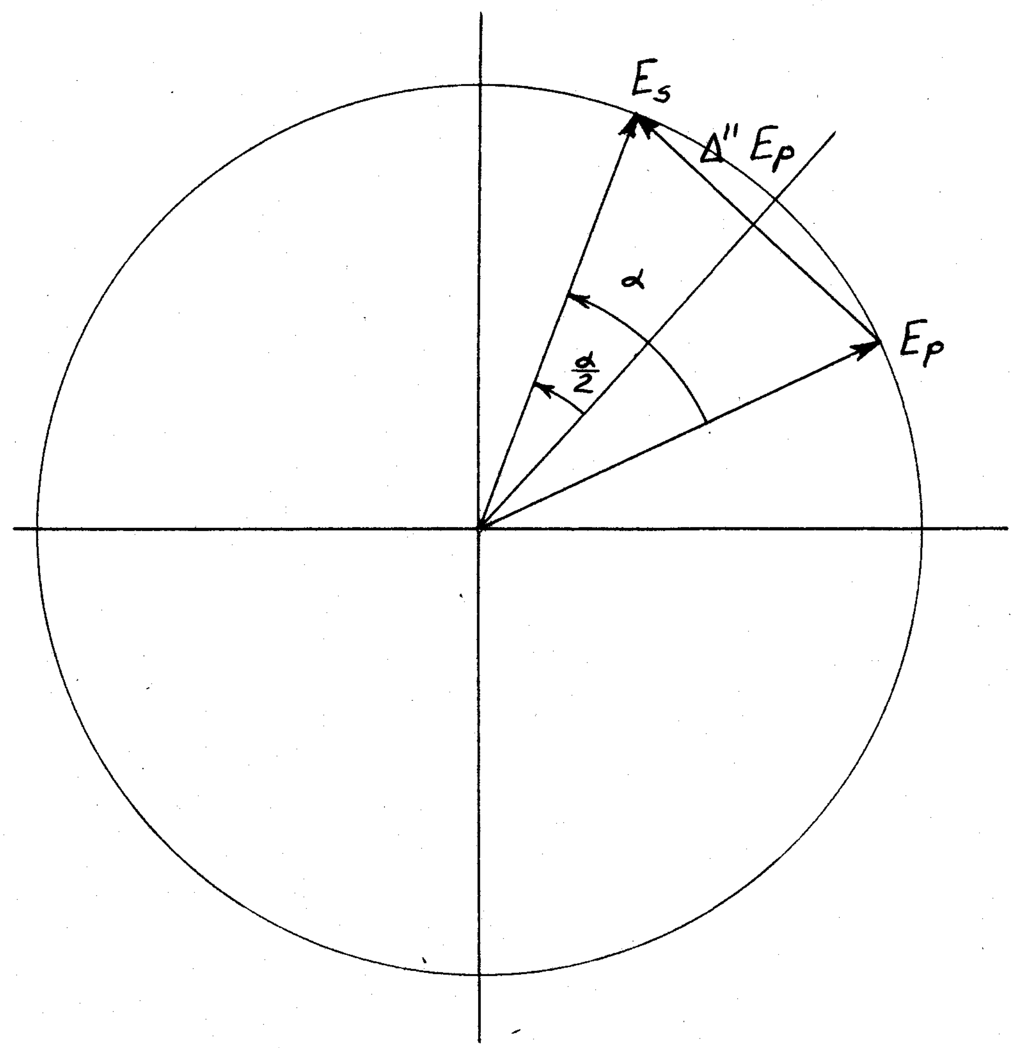


Figure 20. Determination of the Magnitude of Δ''

to relate the phase position of $\underline{\triangle''}$ to the phase transformation constant $\underline{a}$.

The basic requirement of a vector transformation is that the vector power be invariant under the transformation. Thus the sum of the vector powers injected by the generators $\underline{\triangle'' E_p}$ and $\underline{\triangle'' I_p}$ under a phase transformation must be zero.

Referring to Figure 21, it is evident that the quantities of interest may be written in the polar form of complex notation as

$$E_p = |E_p| \; e^{j(\theta + \varphi)}$$

$$I_p = |I_p| \; e^{j\theta}$$

$$\triangle'' E_p = |\triangle'' E_p| \; e^{j(\theta + \varphi + \beta)}$$

$$\triangle'' I_p = |\triangle'' I_p| \; e^{j(\theta + \beta)}$$

$$E_s = |E_p| \; e^{j(\theta + \varphi + a)}$$

$$I_s = |I_p| \; e^{j(\theta + a)} \; .$$

Vector power is given by the product of the conjugate of the complex voltage multiplied by the complex current itself. It is possible then to express the fact that the sum of the power inserted by the current generator and the power inserted by the voltage generator is zero by writing the following equation using the vinculum to denote the complex conjugate.

$$\bar{E}_p \triangle'' I_p + \overline{\triangle'' E_p} \, I_s = 0 \qquad (50)$$

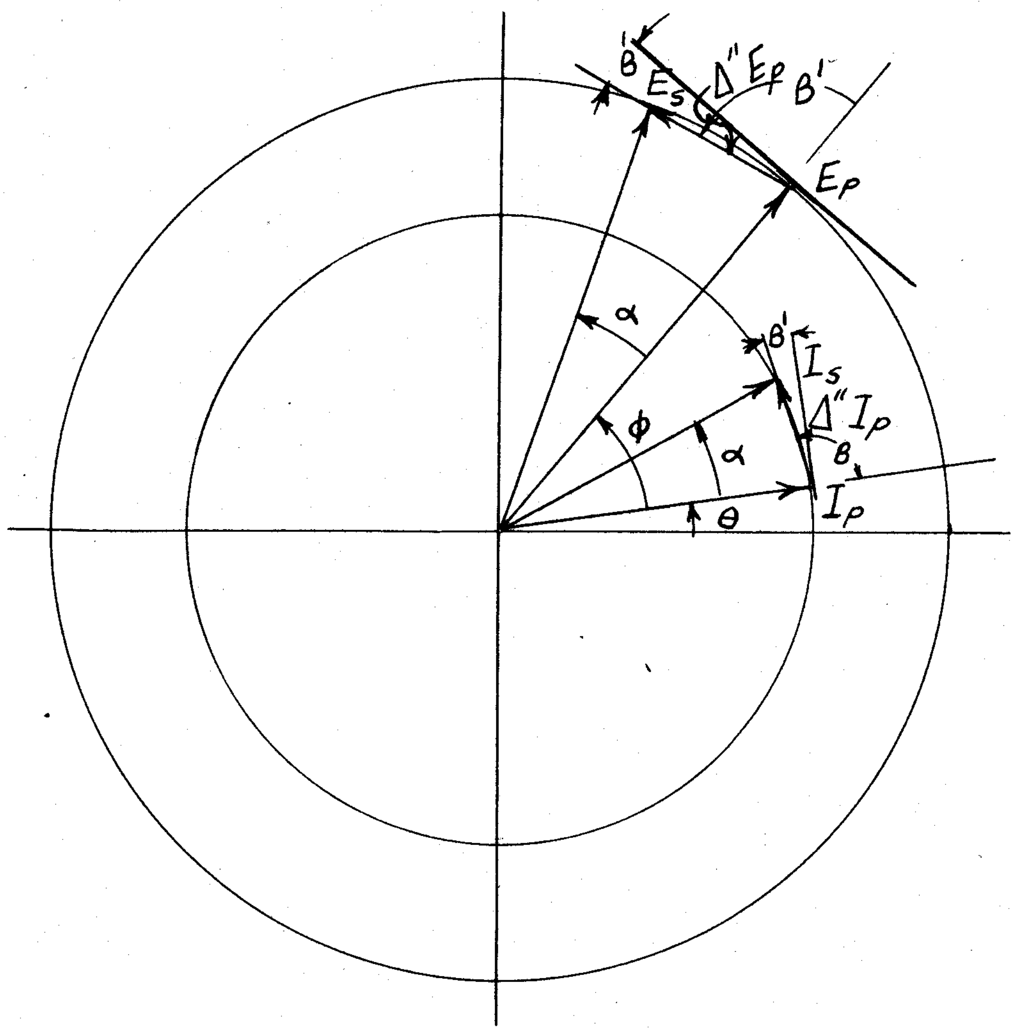


Figure 21. Determination of the Phase Position of Δ"

$$|E_p|e^{-j(\theta + \varphi)}|\Delta''I_p|e^{j(\theta + \beta)} + |\Delta''E_p|e^{-j(\theta + \varphi + \beta)}|I_p|e^{j(\theta + \alpha)} = 0$$

$$|E_p||\Delta''I_p|e^{j(\beta - \varphi)} + |\Delta''E_p|e^{-j(\theta + \varphi + \beta)}|I_p|e^{j(\theta + \alpha)} = 0$$

$$|E_p||\Delta''I_p|e^{j(\beta - \varphi)} = -|\Delta''E_p||I_p|\,e^{j(\alpha - \beta - \varphi)}$$

$$\left[|E_p||\Delta''I_p|\right] e^{j\beta} = \left[|\Delta''E_p||I_p|\right] e^{j(\alpha - \beta + \pi)} \tag{51}$$

Thus,

$$\left[|E_p||\Delta''I_p|\right] = \left[|\Delta''E_p||I_p|\right] \tag{52}$$

And,

$$\beta = \alpha - \beta + \pi \tag{53}$$

$$2\beta = \alpha + \pi$$

$$\beta = \frac{\pi}{2} + \frac{\alpha}{2}$$

$$= 90^\circ + \frac{\alpha}{2} \tag{54}$$

Defining β' as,

$$\beta' = \beta - 90^\circ \tag{55}$$

There results,

$$\beta' = \frac{\alpha}{2} \tag{56}$$

It has therefore been shown that to realize a shift in phase of $\underline{\alpha}$ degrees under a pure angular transformation the magnitude of the vector $\underline{\Delta''}$ must be equal to 2 Sin $\underline{\frac{\alpha}{2}}$ and the phase angle of $\underline{\Delta''}$ must be equal to $\underline{90^\circ + \frac{\alpha}{2}}$.

Thus,

$$\triangle'' = \left| 2 \operatorname{Sin} \frac{\alpha}{2} \right| e^{j\left(\frac{\pi}{2} + \frac{\alpha}{2}\right)} \tag{57}$$

V. CONSTRUCTION OF THE NETWORK ANALYZER REPRESENTATION OF THE PHASE TRANSFORMER

The information which has been given served as the foundation for the synthesis of a phase-shifting device to simulate the effect of phase transformations in the network analyzer representation of circuits involving vector transformers. A block diagram of the type of circuit which was selected appears in Figure 22. The signal voltage for the voltage injection circuit, the appropriate fraction of the terminal input voltage of the simulated transformer, was derived with the aid of a voltage divider. This voltage was then shifted in phase by the amount $\underline{90^\circ + \frac{\alpha}{2}}$ and supplied to an amplifier with a very low output impedance. Such an amplifier closely approaches a voltage source in its characteristics. By means of the voltage divider the amplifier was adjusted to supply a voltage of magnitude $\underline{(2 \sin \frac{\alpha}{2}) E_p}$ at a phase angle of $\underline{90^\circ + \frac{\alpha}{2}}$ with respect to the input voltage. The voltage source was then connected in series with the input voltage $\underline{E_p}$.

Similarly the excitation for the current injection circuit was derived from the input current with the aid of a low resistance shunt and a current transformer. This signal was also advanced in phase by $\underline{90^\circ + \frac{\alpha}{2}}$ and used to excite an amplifier employing pentode output tubes

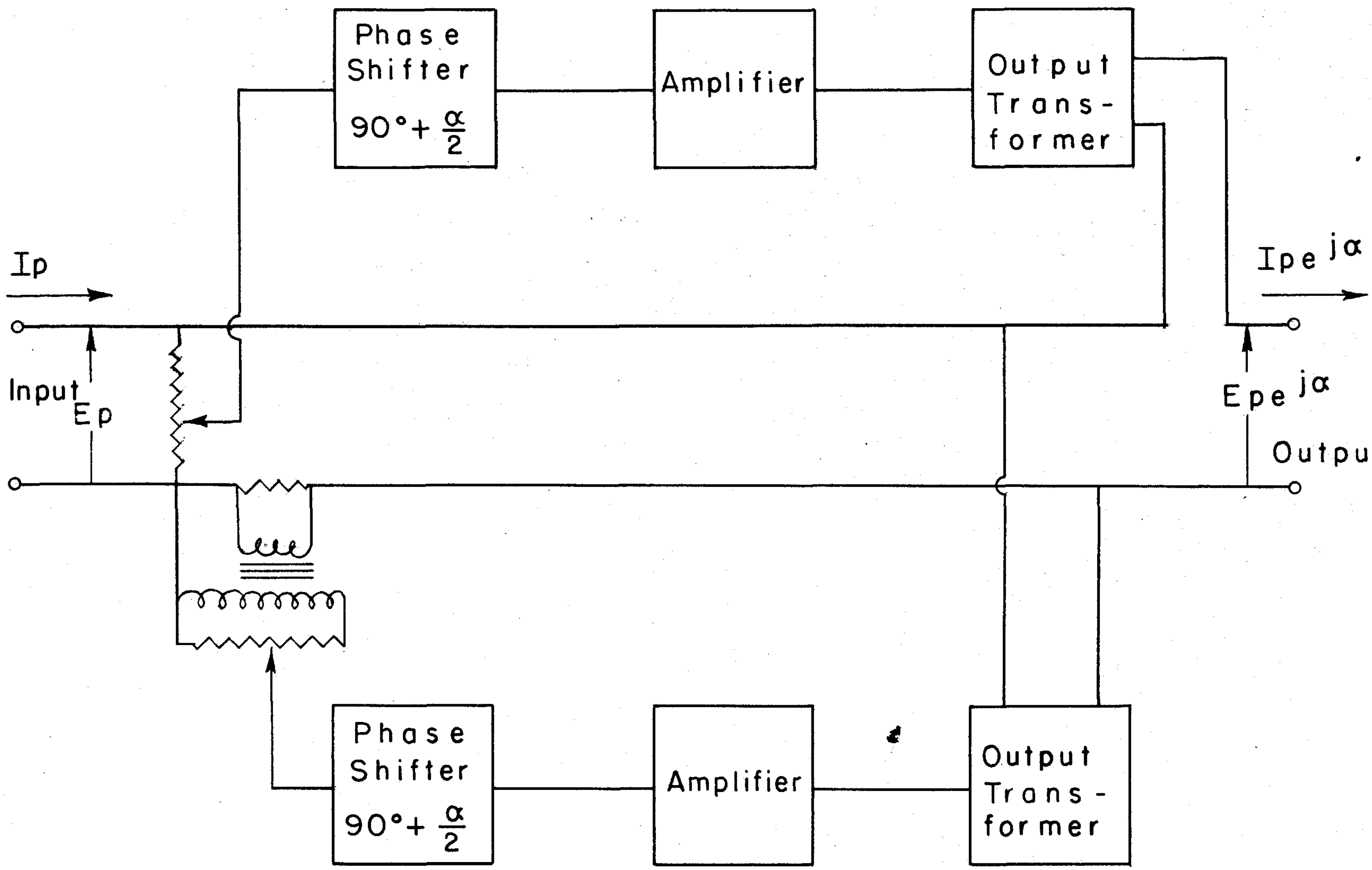


Figure 22. Simplified General Scheme of Two-Generator Phase Transformer

transformer coupled across the input terminals of the simulated phase angle transformer. Such a pentode circuit may be arranged to display an exceedingly high internal impedance, and therefore it closely approximates a current source. The circuit appears in detail in Figure 23, and a detailed account of its construction will now be given.

The input voltage for the voltage injection circuit was derived from the junction point of two resistors $\underline{R_1}$ and $\underline{R_2}$ one of 47,000 ohms and one of 22,000 ohms connected in series across the line. This voltage divider was used in order to keep the grid signal of the subsequent cathode follower stage, $\underline{V_1}$, within the negative region of grid voltage operation. The output resistor of this stage was a 2,000 ohm voltage divider $\underline{R_3}$ in series with a 1,500 ohm voltage divider $\underline{R_4}$. The 2,000 ohm voltage divider was provided for the purpose of adjusting the absolute magnitude of the injected voltage to the required value. The 1,500 ohm voltage divider existed for the purpose of compensating for any voltage changes that occurred incidental to phase adjustments. As these effects were found to be quite small this adjustment was not used in the later operation of the device. The output of this cathode follower supplied energy to a phase shifter of the conventional R-C type with a center tapped transformer, $\underline{T_1}$. As only the angle $\underline{\beta'}$ was to be changed and since it involved but a very small range of angles it was deemed advisable to provide two series controls with the smaller one placed on the front panel to give vernier control of the angle $\underline{\beta'}$. The larger control, $\underline{R_5}$, was mounted on the chassis behind the panel and permanently set to provide a 90° phase shift. The angle $\underline{\beta'}$ was then

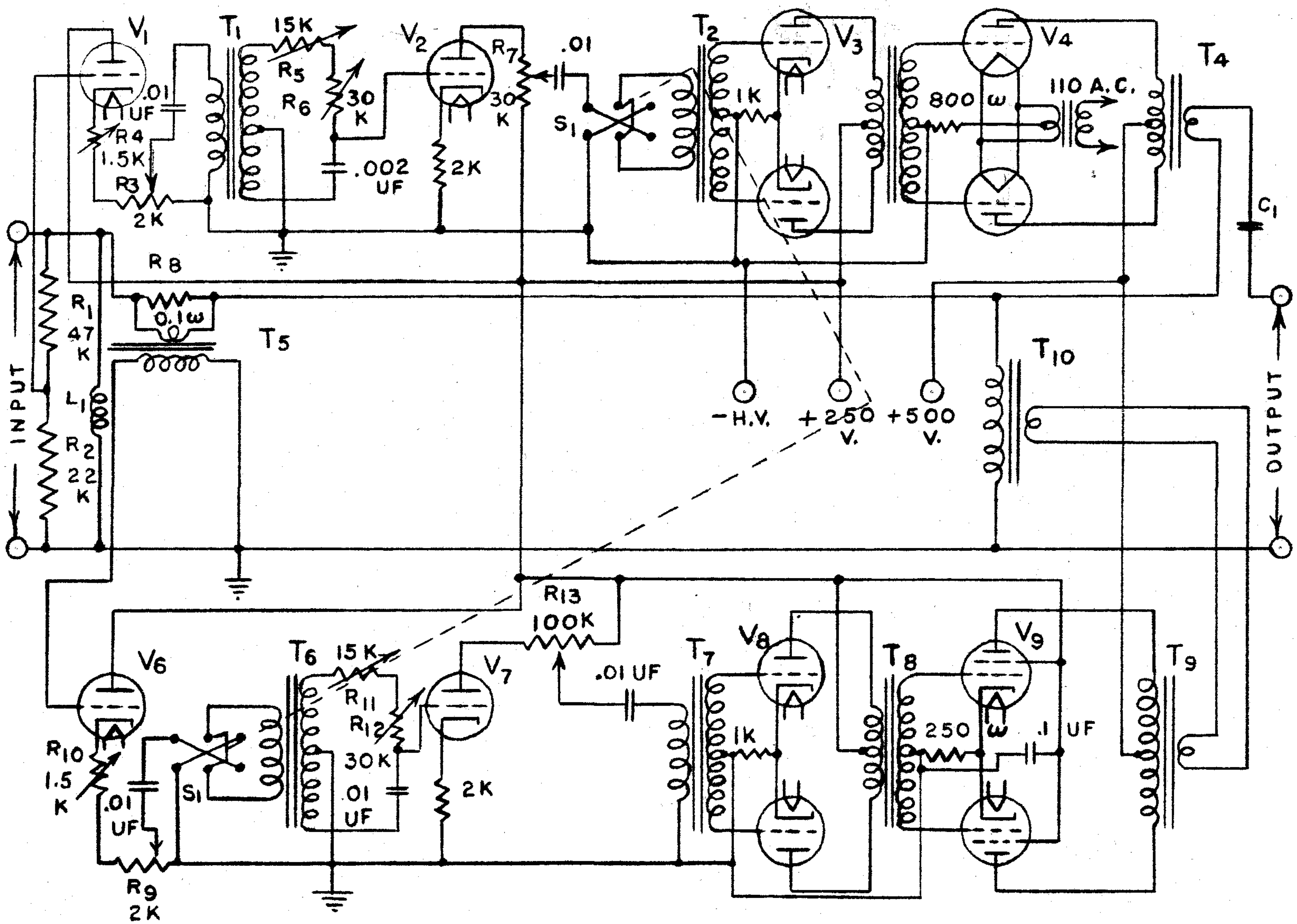


Figure 23. Detailed Circuit Diagram

adjusted to its proper value for a particular operating condition by the vernier control, R_6, on the front panel.

The phase shifter was allowed to excite the grid of an amplifier tube with a 30,000 ohm voltage divider, R_7, in its plate lead. This voltage divider was used to adjust the general voltage level such that a maximum setting of the cathode follower voltage divider would just drive the injection circuit to its maximum capacity. It was found desirable to isolate the phase shift network transformer, T_1, by this use of a cathode follower and an amplifier tube to avoid the loading effects produced by the finite impedances of the transformer. The signal was then passed through a four-pole-double-throw reversing switch, S_1, and transformer coupled to a pair of 6C5's, V_3, which drove a pair of 6B4's, V_4, in push pull. The type 6B4 was selected as this tube has a very low plate resistance. The output transformer, T_4, chosen was a high quality output transformer with an unusually low primary impedance. The transformer was disassembled and its secondary was rewound with fewer turns of larger wire in order to provide as closely as possible the proper transformation ratio with the least possible secondary impedance. The transformer was designed to produce a quadrature voltage sufficient to shift a fifteen volt input potential through fifteen degrees. This is an adequate range to satisfy the phase shifting problems which must be solved on the I. S. C. network analyzer, because the phase transformers which are employed in electric power networks are not ordinarily designed to provide more than 12 degrees of phase shift.[11]

The current injection circuit, by virtue of the symmetry of the situation, was arranged in much the same manner. A current transformer, $\underline{T_5}$, excited by the potential drop across the one-tenth ohm shunt $\underline{R_8}$ was arranged to excite the grid of the cathode follower stage, $\underline{V_6}$. Again the cathode follower is provided with the magnitude adjustment $\underline{R_9}$ which appears on the front panel and the corrector for changes of magnitude with phase angle, $\underline{R_{10}}$. Connection to the output was then made to the reversing switch, $\underline{S_1}$, which fed the phase-shifting circuit. The reversing switch is incorporated in the circuit to allow the injected values to be reversed through 180° in order to make available both positive and negative phase shifts. The phase-shifting network provided the variable resistors $\underline{R_{11}}$ for front panel control of $\underline{\beta'}$ and $\underline{R_{12}}$ for the permanent setting of a 90° phase shift. The phase-shift network was directly connected to the grid of $\underline{V_7}$. $\underline{V_7}$ excites the push-pull driver stage $\underline{V_8}$ through a voltage divider $\underline{R_{13}}$ used for the purpose of setting the general level such that a maximum setting of the magnitude control, $\underline{R_9}$, will just utilize the full capabilities of the circuit. The output stage $\underline{V_9}$ consists of two <u>6L6's</u> connected as push-pull pentodes. The output of this stage was coupled to the input circuit of the simulated transformer by a low impedance link between the transformers $\underline{T_9}$ and $\underline{T_{10}}$. $\underline{T_9}$ and $\underline{T_{10}}$ were two low quality output transformers. $\underline{T_{10}}$ was disassembled and turns were removed from its high voltage side to provide a ratio such that the tubes $\underline{V_9}$ were just able to inject enough current to provide a shift of 15° at a maximum loading of 1/2 ampere. Such an arrangement with pentode connected output tubes enabled the realization of a current source approaching sufficiently closely that of a pure current source for the range of operation over which the device was used.

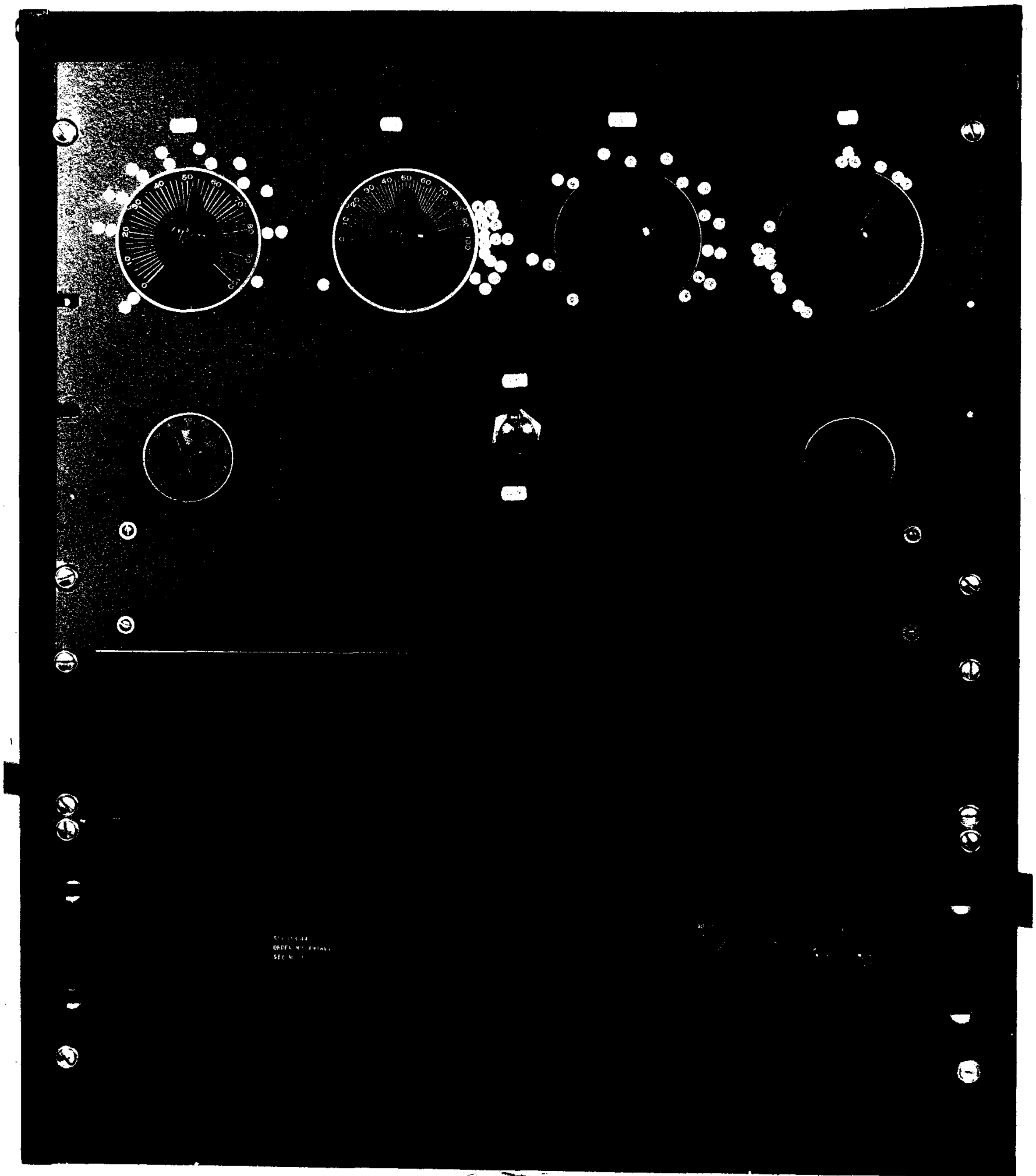
Figure 24 is a front view of the phase transformer which was constructed for use on the Iowa State College 10 kc. Network Analyzer. It is shown mounted in a table rack with the power supply for its low level stages. The 500 volt power supply for the output stages was supplied from an auxiliary source and does not appear in the photograph.

The low-voltage power supply is of the regulated type and is mounted behind the bottom panel of the rack. The phase transformer itself is mounted behind the top panel. The controls of the phase transformer, seven in number, appear on the front of the top panel. At the lower right and lower left are two small knobs. These are auxiliary level controls which may be used to correct for variations of the magnitude of the injected voltage or current which may be incidental to the phase adjustment of these quantities. In practice these variations have been found to be so small that the use of these correctors has not been necessary.

The switch mounted on the lower central portion of the phase transformer panel is for the purpose of reversing the phase of the injected quantities in order to allow the phase transformer to produce negative as well as positive phase transformations.

The four large dials at the top of the panel control the magnitudes and phase angles of the injected quantities. The two on the

Figure 24. Front View of Phase Transformer



left control the current-injection circuit; the two on the right control the voltage-injection circuit.

Of these two groups the right-hand control in each case is for the adjustment of the phase position of the injected quantity, and the left-hand control is for the adjustment of the magnitude of the injected quantity. Small temporary paper markers have been used to provide the calibration for the instrument.

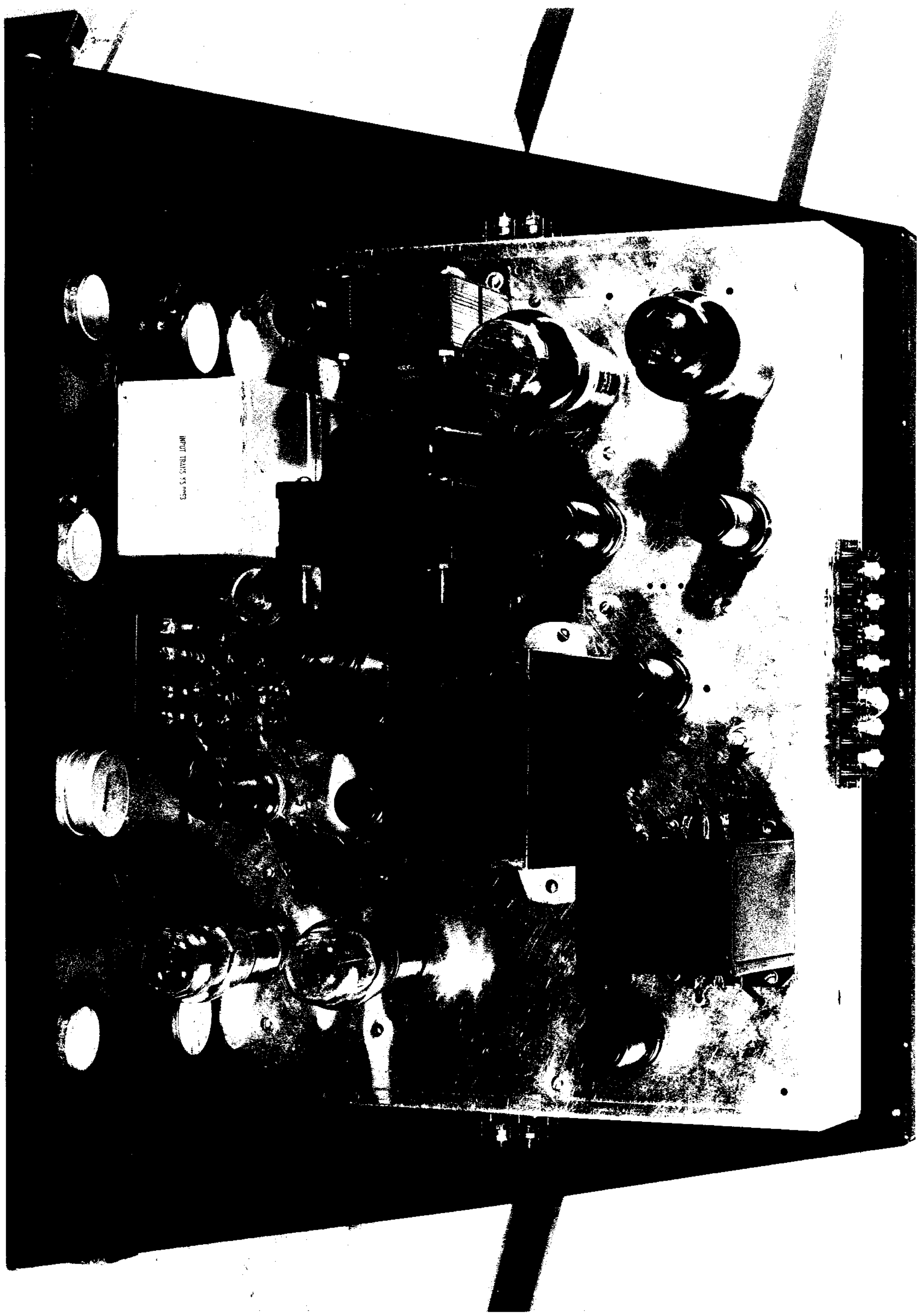
Figure 25 is a rear top view of the phase transformer. This view shows the general placement of the more important parts.

The voltage-injection amplifier appears at the rear of the chassis with the output tubes on the right side in the photograph.

The current-injection amplifier terminates in its output stage shown at the left of the photograph and near the front panel.

The two phase-shifter adjustments for providing 90° of the $90^{\circ} + \beta'$ phase shift required of the injected quantities are also on the top of the chassis. Both are adjustable with the use of a screw driver. The one nearest the front panel adjusts the current-injection circuit.

Figure 25. Rear Top View of Phase Transformer



VI. TEST OF THE TWO GENERATOR EQUIVALENT OF THE PHASE SHIFTING TRANSFORMER

The two generator equivalent of the phase shifting transformer was completed in physical form and when completed was given final tests by connecting it to the network analyzer. By the use of the network analyzer it became possible to observe not only the action of the phase shifter itself, but also to observe the effect of phase transformations throughout the entire simulated system.

The phase shifter had been calibrated crudely previous to the test. It was found that even these crude markings were surprisingly useful in setting up the phase shifter for a given angle of transformation. The two generator equivalent of the phase transformer was tested under many different conditions of loading with the particular objective in mind of observing any tendency toward instability which might exist. No instability was observed in these tests for all transformation angles up to and including fifteen degrees.

The equivalent series resistance and reactance of the phase shifter as well as its apparent shunt conductance and susceptance was determined by network analyzer readings. These values of reactance and susceptance given below are those net values which remain after an effort has been made to tune certain of these quantities out by the use of suitable reactances.

These final values for the device appear in Table I.

TABLE I

Phase Transformer Constants

Series resistance	0.692 ohm
Series reactance	+0.355 ohm
Series impedance	0.781 ohm
Shunt conductance	0.00012 mho
Shunt susceptance	+0.000014 mho
Shunt admittance	.00012 mho

The phase transformer was then connected into a simple typical loop circuit. Readings were taken at all points in the circuit. The circuit chosen was that shown in Figure 26. The circuit behavior has been computed and a tabulation of the circuit conditions as given by analyzer study and by direct computation can be found in Table III of the appendix.

It was found possible to adjust the phase transformer by virtue of its active sources to produce a given phase transformation without <u>any</u> loss of power either real or reactive within the device even in the presence of the imperfections noted in Table I. This adjustment was carried out by manually adjusting each control of the phase transformer until the terminal conditions required of a perfect transformer were met. Under these circumstances the net power consumed by the device, both real and reactive, was actually measured and found to be

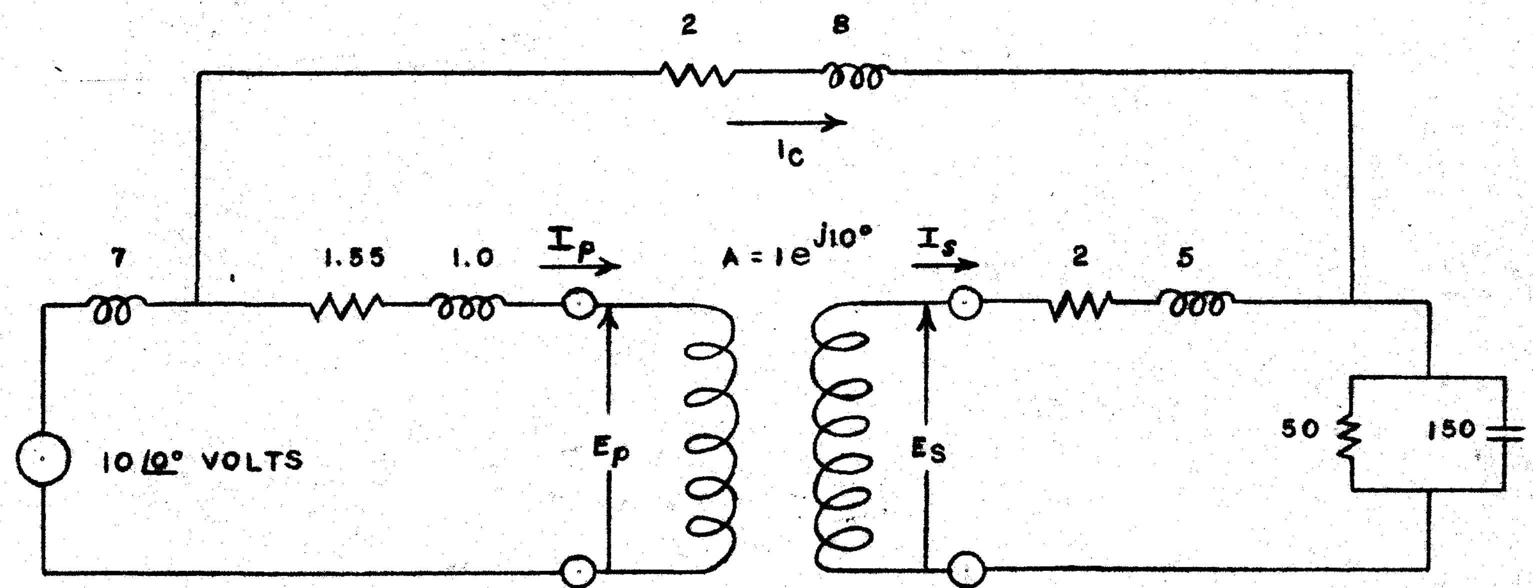


Figure 26. Trial Circuit

equal to zero.

The experimental work was concluded by running the curves shown in Figure 27 and Figure 28 taken from the data of Table II.

These curves illustrate the regulation of the device.

The regulation curves of Figures 27, and 28 have been plotted in order to display the regulation characteristics of the phase transformer. These regulation curves were determined by adjusting the phase transformer controls to produce a perfect transformation ratio of +10 degrees while the phase transformer was loaded with a pure resistance load drawing one-half of the rated maximum current of the analyzer. Then with the controls of the phase transformer untouched its performance was measured under a series of loads from zero to full load with pure resistance loads, with pure inductance loads, and with pure capacitance loads.

The curves of Figure 28 have been plotted to an expanded vertical scale in order to display more clearly the effect of this loading on the ratios of the current magnitudes and on the ratios of the voltage magnitudes at the input and output terminals under various loads.

TABLE II

Data and Derived Values for Regulation Curves

In	Out	Current Amperes	Voltage Volts	$\frac{V_s}{V_p}$	$\frac{I_s}{I_p}$	α_V	α_I	Load†
*		$0.012 \angle{-18^\circ}$	$10.15 \angle{1.0^\circ}$	1.02	---	10.5°	---	O
	*	0.000	$10.35 \angle{11.5^\circ}$					
*		$0.126 \angle{0.0^\circ}$	$9.85 \angle{-1.0^\circ}$	1.015	0.993	10.5°	10.5°	R
	*	$0.125 \angle{10.5^\circ}$	$10.0 \angle{9.5^\circ}$					
*		$0.250 \angle{0.0^\circ}$	$10.0 \angle{0.0^\circ}$	1.00	1.002	10.0°	10.0°	R
	*	$0.251 \angle{10.0^\circ}$	$10.0 \angle{10.0^\circ}$					
*		$0.373 \angle{1.0^\circ}$	$10.05 \angle{0.0^\circ}$	0.985	1.008	10.0°	9.5°	R
	*	$0.375 \angle{10.5^\circ}$	$10.0 \angle{10.0^\circ}$					
*		$0.492 \angle{1.0^\circ}$	$10.15 \angle{0.0^\circ}$	0.976	1.015	10.0°	9.0°	R
	*	$0.500 \angle{10.0^\circ}$	$10.0 \angle{10.0^\circ}$					
*		$0.125 \angle{-79.0^\circ}$	$9.85 \angle{5.0^\circ}$	1.015	1.00	10.5°	9.5°	L
	*	$0.125 \angle{-69.5^\circ}$	$10.0 \angle{15.5^\circ}$					
*		$0.248 \angle{-74.5^\circ}$	$9.85 \angle{8.5^\circ}$	1.015	1.008	11.5°	9.5°	L
	*	$0.250 \angle{-65.0^\circ}$	$10.0 \angle{20.0^\circ}$					
*		$0.370 \angle{-71.5^\circ}$	$9.9 \angle{12.0^\circ}$	1.01	1.013	12.0°	9.0°	L
	*	$0.375 \angle{-62.5^\circ}$	$10.0 \angle{24.0^\circ}$					
*		$0.486 \angle{-67.5^\circ}$	$9.93 \angle{15.5^\circ}$	1.008	1.018	12.0°	8.0°	L
	*	$0.495 \angle{-59.5^\circ}$	$10.0 \angle{27.5^\circ}$					
*		$0.123 \angle{88.5^\circ}$	$9.73 \angle{-2.0^\circ}$	1.022	1.025	9.0°	10.0°	C
	*	$0.126 \angle{98.5^\circ}$	$9.95 \angle{7.0^\circ}$					
*		$0.245 \angle{84.5^\circ}$	$9.73 \angle{-6.0^\circ}$	1.027	1.020	9.0°	9.5°	C
	*	$0.250 \angle{94.0^\circ}$	$10.0 \angle{3.0^\circ}$					
*		$0.370 \angle{79.5^\circ}$	$9.68 \angle{-9.5^\circ}$	1.032	1.013	8.0°	9.0°	C
	*	$0.375 \angle{88.5^\circ}$	$10.0 \angle{-1.5^\circ}$					
*		$0.493 \angle{74.0^\circ}$	$9.65 \angle{-14.0^\circ}$	1.036	1.013	7.5°	9.5°	C
	*	$0.500 \angle{83.5^\circ}$	$10.0 \angle{-6.5^\circ}$					

† The designations O, R, L, and C denote no load, pure resistance load, pure inductance load, and pure capacitance load respectively.

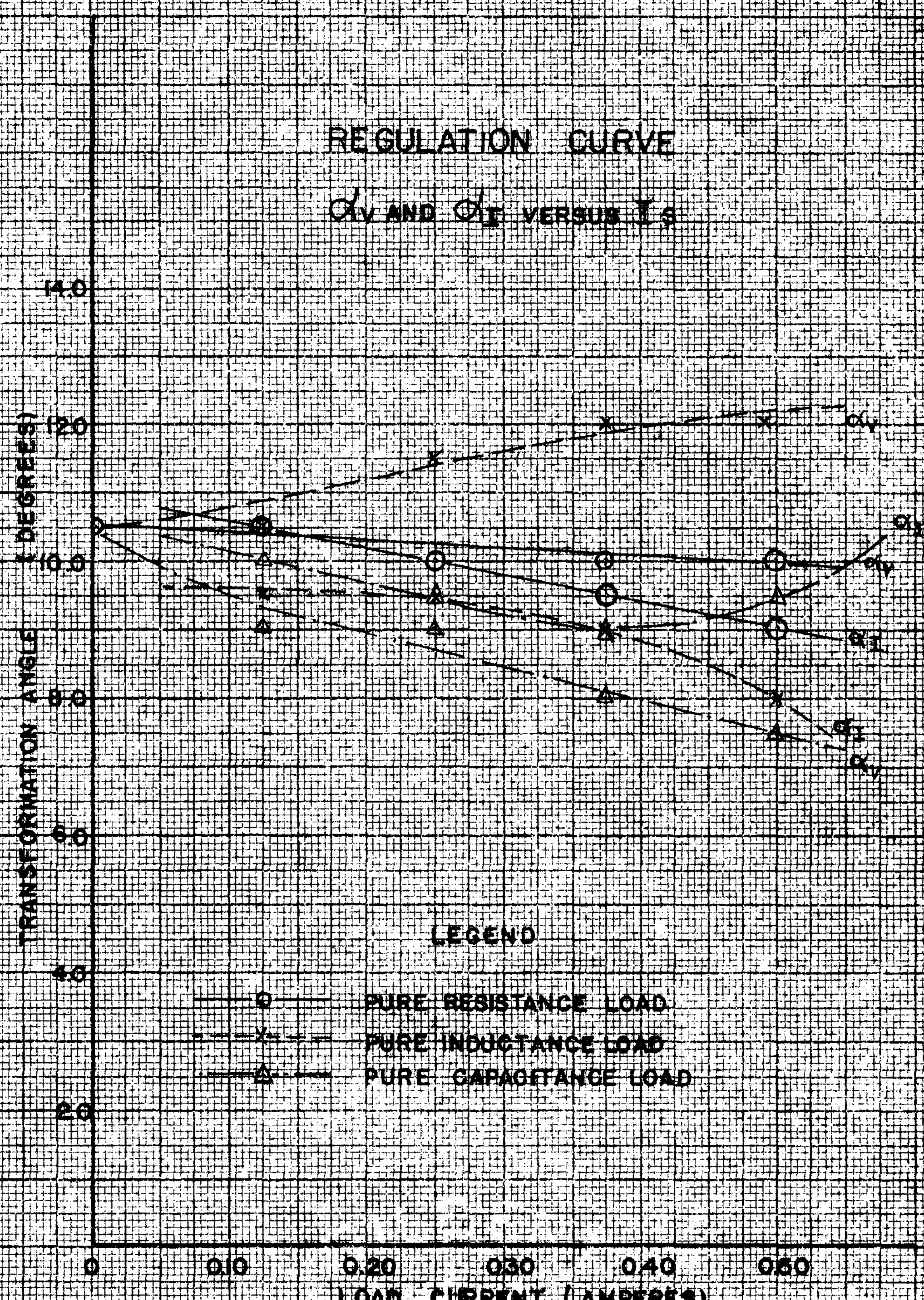


Figure 27. Phase Regulation Curves

Form E-3

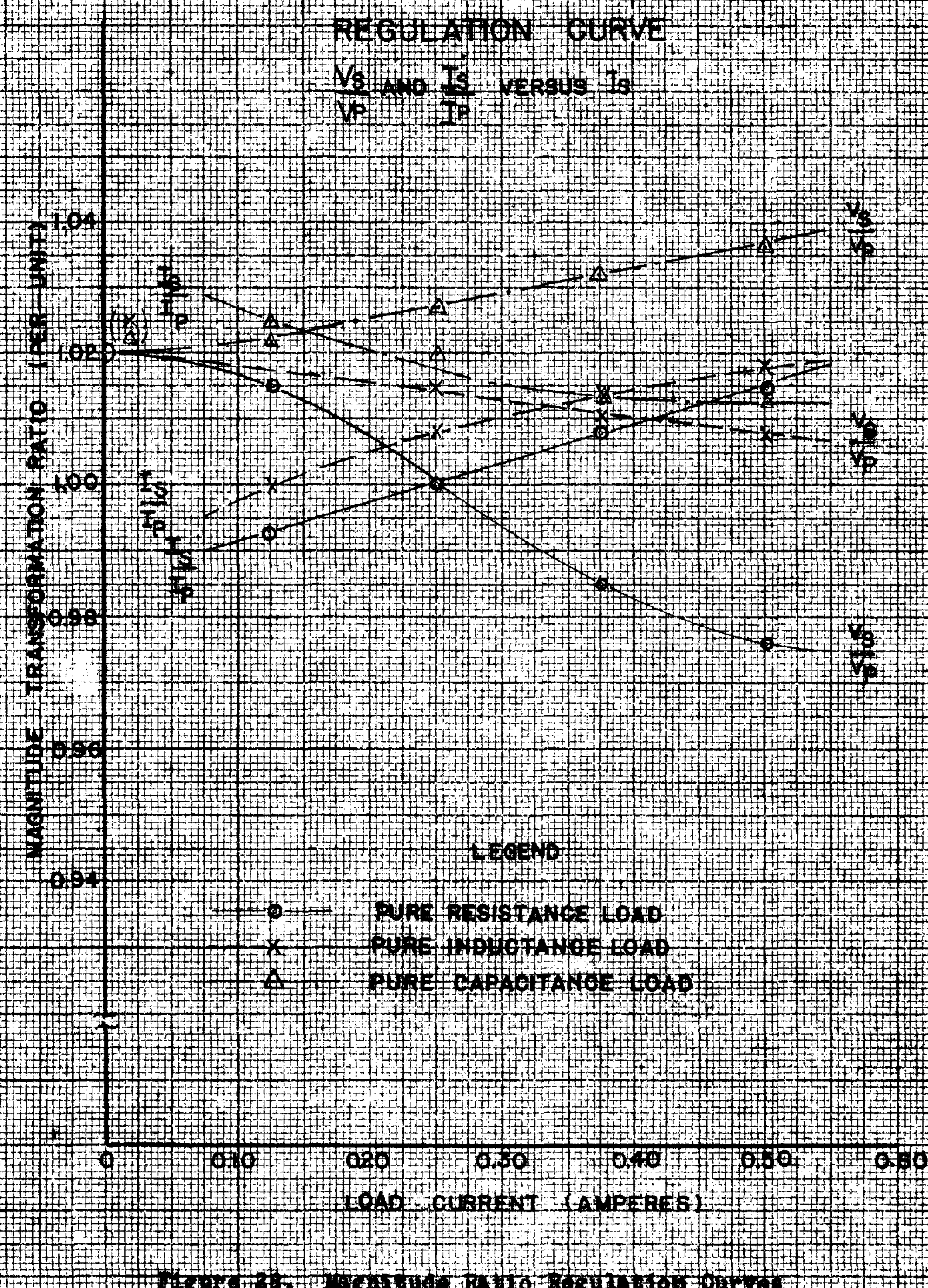


Figure 28. Magnitude Ratio Regulation Curves

VII. DISCUSSION

The two generator equivalent circuit of the vector transformer has been shown to be adaptable to both analytic and analyzer studies of loop systems.

As an analytic instrumentality it has the advantage that it utilizes the base-ratio solution of the network which is generally already known, or if not known, may be obtained by the comparatively easy method of reduction to a common base. The solution then is simply obtained by superimposing the effect of the two equivalent generators on the system. The solution may be obtained for a general vector transformation almost as easily as for the special cases of either a pure magnitude or a pure phase transformation. Thus the method is not limited in generality either by theoretical or practical considerations.

The solution obtained is an exact one although approximations are always evident which will greatly decrease the labor of solution should it be elected that they be employed.

The current generated by the current generator is of considerable interest. This current is given by $\underline{(1 - \frac{e^{j2\alpha}}{A})I_p}$ for a general vector transformation and by $\underline{-(\frac{\Delta'}{1 + \Delta'})I_p}$ for a magnitude transformation. Occasionally it is desirable to omit the current generator in an approximate evaluation of the operation of a circuit. It will be observed that the relative contribution of the current generator is given by the fraction,

$$\delta' = -\left(\frac{\triangle'}{1 + \triangle'}\right). \tag{31}$$

But for a magnitude transformation, from (35),

$$\triangle' = a - 1$$

so

$$\delta' = -\left(\frac{a - 1}{a}\right)$$

$$= \left(\frac{1 - a}{a}\right). \tag{58}$$

This fraction is plotted as a function of the scalar transformation ratio, a, in Figure 27, and it will serve as some indication in the case of scalar transformations of the relative effect of the shunt generator on system currents.

The physical representation of the phase transformer by the two-generator method has shown itself to be of particular value in network analyzer studies in which heretofore no satisfactory technique for the purpose has existed. The two-generator phase shifter possesses the property of being capable of being set to any given phase angle within its range and retaining closely this setting under all loads and operating conditions within the capabilities of the device. These benefits accrue partially from the electronic circuitry that is associated with the two generators. Unfortunately the phase settings are not retained perfectly under all loads for the two generator

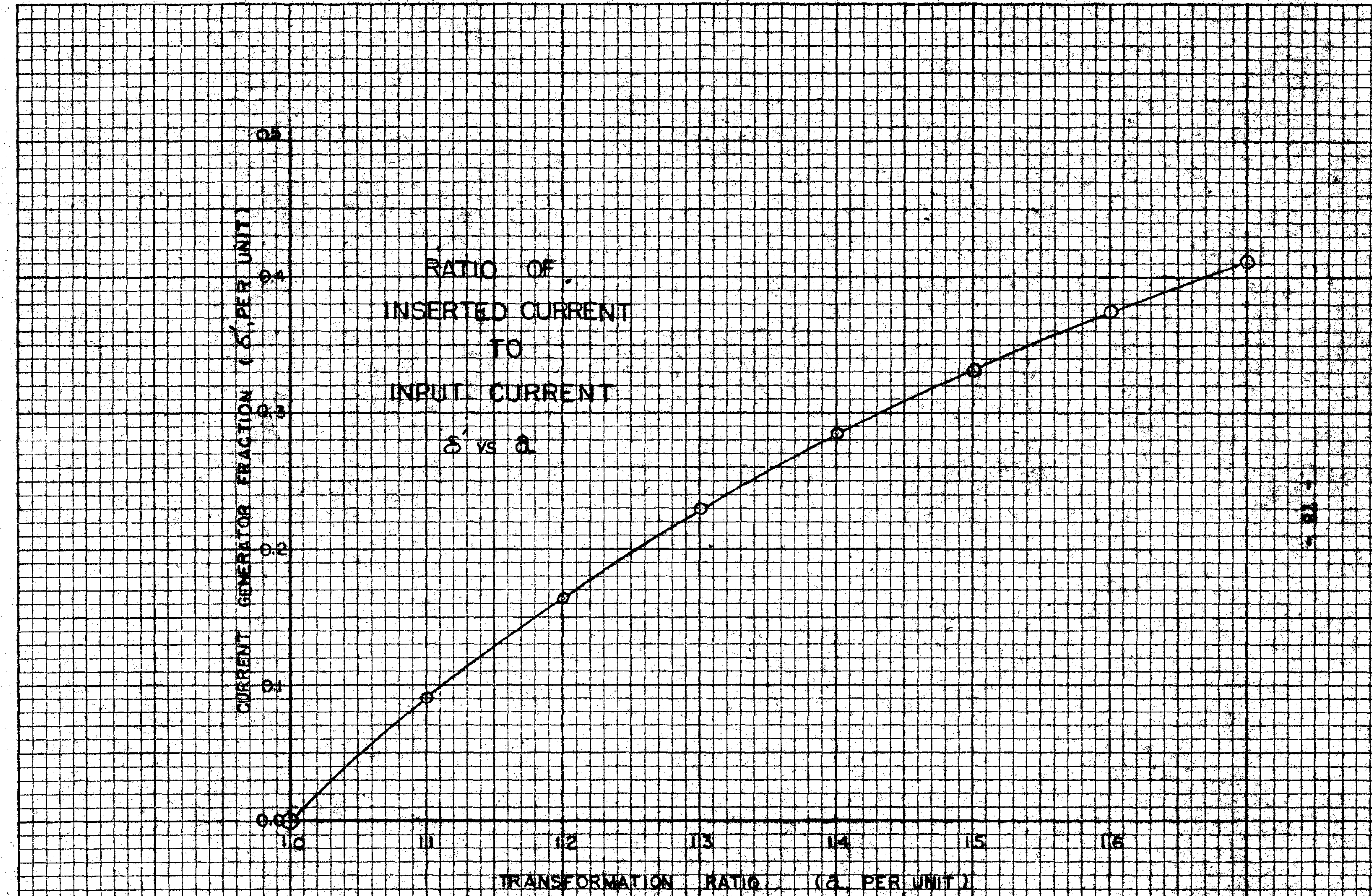


Figure 29. Curve of ' vs. a

equivalent is itself a physical device with attendant internal impedance and internal shunt admittance. It must not be forgotten, however, that the two generator equivalent circuit will be used to represent other physical devices and that these devices always have internal series impedance and shunt admittance also.

In the event it becomes necessary to simulate the performance of a perfect transformer this, too, is within the capabilities of the two generator method because the active sources may, by trifling individual adjustments of their magnitudes and phase angle, be made to compensate for their internal losses.

The adjustments required are fairly independent of each other as the adjustments for magnitude and phase are mutually orthogonal to each other in the complex plane. Thus it is possible to make each adjustment in turn and find upon completing the four adjustments that each is essentially as it was first set and has not been influenced appreciably by later adjustments.

The internal series resistance of the series generator of the two generator equivalent depends directly upon the secondary winding resistance of the output transformer and the sum of the primary winding resistance and the tube plate-to-plate resistances both of the latter divided by the square of the transformation ratio of the output transformer. Thus if large incremental voltages are to be injected the value of the transformation constant will be low and there will have to be many turns on the secondary winding of the output transformer. This means that the equivalent series impedance will be high. The smaller the required phase transformation angle the smaller will be the required voltage which must be injected and the higher the transformation ratio of the output transformer. It would be very desirable to have taps on the output transformer in order that the lowest series insertion impedance possible be inserted for each phase angle transformation. This would be a modification that should be considered if insertion impedance is an important factor in a particular application of the device.

Similar remarks may be made for the current generator. In the case of the current generator it is essential that a low transformation ratio be maintained in order that the shunt admittance of the generator be maintained as low as possible. Again for the smaller phase angle transformation ratios the transformation ratio of the output transformer may be much lower thereby producing a much lower shunt conductance than is possible for the larger phase angle transformation ratios. Again, if shunt conductance proves important, the characteristics of the device can be improved by the use of a tap changer on the secondary winding of the output transformer of the current generator.

The front panel control of the angle $\underline{\beta'}$ may in general be found inadequate for adjusting for a lossless transformation under heavy load as this adjustment allows the angle $\underline{\beta'}$ to be varied by only a few degrees which may be insufficient to allow the introduction of sufficient energy to counteract the losses under load. The larger angle $\underline{\beta}$ is adjustable by a potentiometer at the rear of the chassis. Normally this control is set to provide a 90° phase shift, the additional small angle is provided by the $\underline{\beta'}$ control on the front panel. If a means is provided for accurately resetting the $\underline{\beta}$ control to 90° there is no objection to altering its setting to simulate a perfect transformation.

Negative transformation angles are quite as likely to be encountered on transmission systems as positive angles. For this reason it is just as important that negative angles be representable as it is that positive angles be representable.

Negative sequence quantities are shifted negatively by the transformation angle in passing through a phase shifting transformer. The solution of unbalanced networks by the method of symmetrical components requires the solution of both the positive and negative sequence networks. Thus it is necessary to provide for both positive and negative phase angle shifts of equal amplitude in determinations of this kind.

VIII. SUMMARY

It has been shown here that a perfect vector transformer may be replaced by a perfect current generator and a perfect voltage generator with no change in terminal conditions. This resulting equivalent circuit has been demonstrated to provide a method by which to attack transformer problems. For the general case of loop circuits in which the transformation ratios fail to form a product of real unity around the loop the analytical approach this method provides to the exact solution is simpler than the classical method of solution.

A phase transformer based upon this equivalent circuit has been constructed. This device has proved very useful for representing phase transformations on a network analyzer.

A sample solution has been presented of a particular problem to demonstrate the relative merits of the classical solution, the solution by the two generator equivalent representation, and the solution by means of the network analyzer utilizing the physical counterpart of the two-generator equivalent of the phase transformer.

The solutions by these three methods have been found to be in proper agreement.

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X. ACKNOWLEDGMENTS

The author wishes to express his appreciation to Dr. W. B. Boast for the original suggestion of this topic and for his guidance and counsel throughout the entire investigation.

Thanks are also extended to Professor W. L. Cassell for early supervision of the writer's research program, and to Professor M. S. Coover for creating an atmosphere so very agreeable to the research student.

The author wishes to thank Mr. C. E. Otis, Mr. P. A. Kristensen, and Miss Beulah Lande for aid most abundantly given.

XI. APPENDICES

APPENDIX A. SOLUTION OF A TYPICAL LOOP CIRCUIT BY THE CLASSICAL METHOD

An example will now be given of the solution by the classical method of a typical circuit involving a pure phase transformation of $+10^\circ$. The circuit is shown in detail in Figure 30. The loop equations and transformer equations are,

$$V_1 - Z_a I_1 - Z_p(I_1 - I_3) - E_1 = 0 \tag{59}$$

$$E_1 - Z_p(I_3 - I_1) - Z_b(I_3) - Z_s(I_3 - I_2) - E_2 = 0 \tag{60}$$

$$E_2 - Z_s(I_2 - I_3) - Z_c I_2 = 0 \tag{61}$$

$$E_2 = ae^{j\alpha}E_1 \tag{62}$$

$$(I_2 - I_3) = e^{j\alpha}\,\frac{(I_1 - I_3)}{a}\,. \tag{63}$$

These equations form the simultaneous set of equations,

$$I_1(Z_a + Z_p) + I_2(0) + I_3(-Z_p) + E_1(1) + E_2(0) = V_1 \tag{64}$$

$$I_1(-Z_p) + I_2(-Z_s) + I_3(Z_p + Z_s + Z_b) + E_1(-1) + E_2(1) = 0 \tag{65}$$

$$I_1(0) + I_2(Z_s + Z_c) + I_3(-Z_s) + E_1(0) + E_2(-1) = 0 \tag{66}$$

$$I_1(0) + I_2(0) + I_3(0) + E_1(ae^{j\alpha}) + E_2(-1) = 0 \tag{67}$$

$$I_1(1) + I_2(-ae^{-j\alpha}) + I_3(ae^{-j\alpha}-1) + E_1(0) + E_2(0) = 0 \tag{68}$$

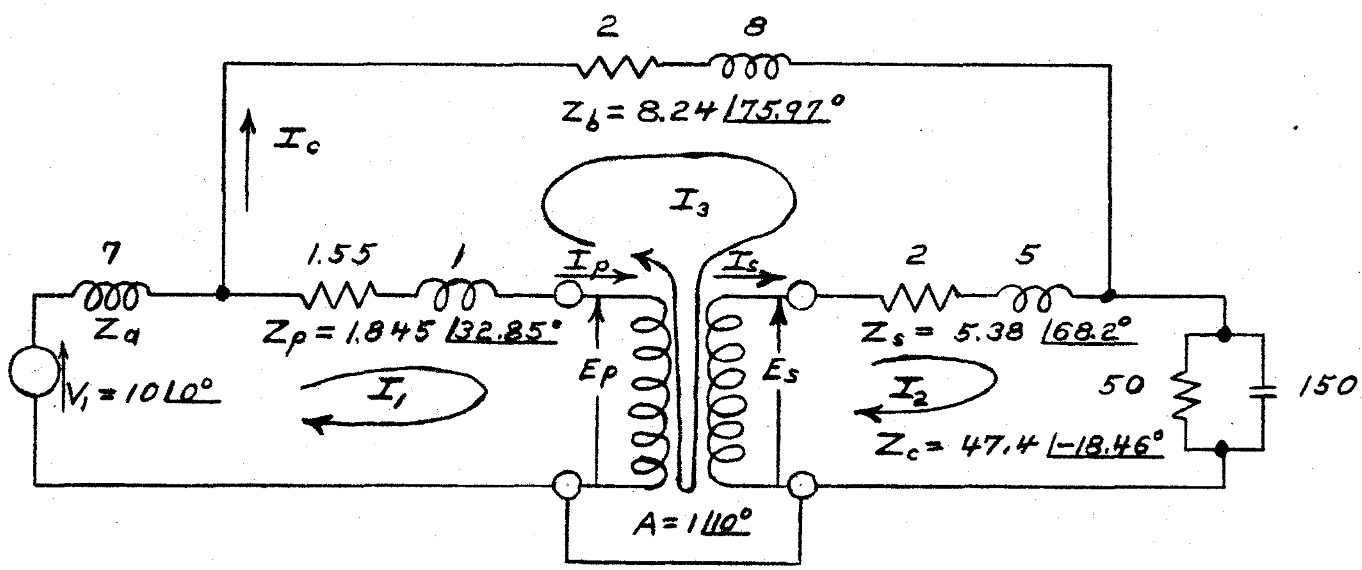

Figure 30. Diagram of Test Circuit

Solving by Cramer's rule the system determinant is,

$$D = \begin{vmatrix} (Z_a + Z_p) & 0 & -Z_p & 1 & 0 \\ -Z_p & -Z_s & (Z_p + Z_s + Z_b) & -1 & 1 \\ 0 & (Z_s + Z_c) & -Z_s & 0 & -1 \\ 0 & 0 & 0 & ae^{j\alpha} & -1 \\ 1 & -ae^{-j\alpha} & (ae^{-j\alpha}-1) & 0 & 0 \end{vmatrix}.$$

Solving the determinant there is obtained,

$$\begin{aligned} D = \; & a^2(Z_aZ_c + Z_pZ_c + Z_aZ_p + Z_aZ_b + Z_pZ_b) \\ & -ae^{j\alpha}(Z_aZ_c) - ae^{-j\alpha}(Z_aZ_c) + (Z_sZ_b + \\ & \quad Z_cZ_s + Z_cZ_b + Z_aZ_s + Z_aZ_c). \end{aligned} \tag{69}$$

For the circuit with the constants shown on Figure 28 the numerical value of the determinant becomes,

$$\begin{aligned} D = \; & (7\underline{/90})(47.4\underline{/-18.46}) + (1.845\underline{/32.85})(47.4\underline{/-18.46}) \\ & + (7\underline{/90})(1.845\underline{/32.85}) + (7\underline{/90})(8.24\underline{/75.97}) \\ & + (1.845\underline{/32.85})(8.24\underline{/75.97}) - (1\underline{/10})(7\underline{/90})(47.4\underline{/-18.46}) \\ & - (1\underline{/-10})(7\underline{/90})(47.4\underline{/-18.46}) + (5.38\underline{/68.2})(8.24\underline{/75.97}) \\ & + (47.4\underline{/-18.46})(5.38\underline{/68.2}) + (47.4\underline{/-18.46})(8.24\underline{/75.97}) \\ & + (7\underline{/90})(5.38\underline{/68.2}) + (7\underline{/90})(47.4\underline{/-18.46}) \\ = \; & 326 + j634.81 \\ = \; & 714\underline{/62.8} \end{aligned} \tag{70}$$

as evaluated with the aid of a slide rule.

The current I_1 is given by,

$$I_1 = \frac{\begin{vmatrix} V_1 & 0 & -Z_p & 1 & 0 \\ 0 & -Z_s & (Z_p + Z_s + Z_b) & -1 & 1 \\ 0 & (Z_s + Z_c) & -Z_s & 0 & -1 \\ 0 & 0 & 0 & ae^{j\alpha} & -1 \\ 0 & -ae^{-j\alpha} & (ae^{-j\alpha}-1) & 0 & 0 \end{vmatrix}}{D}$$

$$= \frac{10\underline{/0}\left[a^2(Z_p + Z_b) - ae^{-j\alpha}(Z_c) - ae^{j\alpha}Z_c + a^2Z_c + (Z_s + Z_c)\right]}{D} \qquad (71)$$

$$DI_1 = 10\underline{/0}\left[(1.55 + j1 + 2 + j8) - (1\underline{/-10})(47.4\underline{/-18.46}) - (1\underline{/10})(47.4\underline{/-18.46}) + (45 - j15) + (2 + j5 + 45 - j15)\right]$$

$$= (70.5 + 135.7)$$

$$= 153.1\ \underline{/62.53}$$

$$I_1 = \frac{153.1\ \underline{/62.53}}{714\ \underline{/62.8}}$$

Hence,

$$I_1 = .217\ \underline{/-0.3}$$

$$= .217 - j0.001136. \qquad (72)$$

The current I_s is given by,

$$I_s = \frac{\begin{vmatrix} (Z_a + Z_p) & V_1 & -Z_p & 1 & 0 \\ -Z_p & 0 & (Z_p + Z_s + Z_b) & -1 & 1 \\ 0 & 0 & -Z_s & 0 & -1 \\ 0 & 0 & 0 & ae^{j\alpha} & -1 \\ 1 & 0 & (ae^{-j\alpha}-1) & 0 & 0 \end{vmatrix}}{D}$$

$$= \frac{V_1\left[a^2 Z_p + ae^{j\alpha} Z_b + Z_s\right]}{D} \qquad (73)$$

For the circuit shown I_s becomes numerically,

$$I_s = \frac{10\angle 0\left[(1.55 + j1) + (1\angle 10)(8.24\angle 75.97) + (2 + j5)\right]}{714\angle 62.8}$$

Hence,

$$I_s = \frac{148.0\angle 73.78}{714\angle 62.8}$$

$$= .2072\angle 10.98$$

$$= .2033 - j0.03944 \; . \qquad (74)$$

The current I_3 is given by,

$$I_3 = \frac{\begin{vmatrix} (Z_a + Z_p) & 0 & V_1 & 1 & 0 \\ -Z_p & -Z_s & 0 & -1 & 1 \\ 0 & (Z_s + Z_c) & 0 & 0 & -1 \\ 0 & 0 & 0 & ae^{j\alpha} & -1 \\ 1 & -ae^{-j\alpha} & 0 & 0 & 0 \end{vmatrix}}{D}$$

$$= \frac{V_1\left[a^2 Z_p - ae^{j\alpha} Z_c + (Z_s + Z_c)\right]}{D} \tag{75}$$

Numerically,

$$I_3 = \frac{10\angle 0\left[(1.55 + j0) - (1\angle 10)(47.4\angle -18.46) + (2+j5) + (45-j15)\right]}{714\angle 62.8}$$

$$= \frac{262\angle -50.9}{714\angle 62.8}$$

$$= .0367\angle -113.7$$

$$= -0.0367\angle 66.3$$

$$= -0.01495 - j0.02408 \tag{76}$$

It is evident from Figure 30 that,

$$I_p = I_1 - I_3 \tag{77}$$

$$I_c = +I_3 \tag{78}$$

$$I_s = I_2 - I_3. \tag{79}$$

Numerically,

$$I_p = (.217 - j0.001136) - (0.01512 + j0.03375)$$

$$= .232 + j0.033$$

$$= .235 \underline{/8} \tag{80}$$

$$I_o = -.037 \underline{/65.9} \tag{81}$$

$$I_s = (.2033 + j0.03944) - (-.01512 - j0.03375)$$

$$= .2184 + j0.0732$$

$$= .231 \underline{/+18.52} \quad . \tag{82}$$

The currents I_1, I_2 and I_3, and I_p, I_o and I_s are the quantities of greatest interest here. Their values will now be computed by a second method.

APPENDIX B. SOLUTION OF A TYPICAL LOOP CIRCUIT BY THE TWO-GENERATOR METHOD

The two-generator method for the analytical solution of problems involving vector transformations will now be illustrated by means of an example. Again, the circuit chosen for the example is that shown in Figure 30.

The base-ratio circuit is shown in Figure 31. The impedance Z_{bo} shown on Figure 31 is,

$$Z_{bo} = \frac{(8.24\angle 75.97)(6.98\angle 59.4)}{(2+j8) + (3.55+j6)}$$

$$= 1.492 + j3.522 \ .$$

The reduced circuits of Figures 32-A and 32-B may then be drawn. The base-ratio generator current, I_{go} is then evidently,

$$I_{go} = \frac{10\angle 0}{44.6\angle -5.33} \qquad (83)$$

$$= .2145\angle 5.53$$

$$= .2137 + j.02036$$

The voltage V_{bo} is,

$$V_{bo} = I_{go}Z_{bo}$$

$$= (3.825\angle 67.02)(.2145\angle 5.53)$$

$$= .8185\angle 72.55 \qquad (84)$$

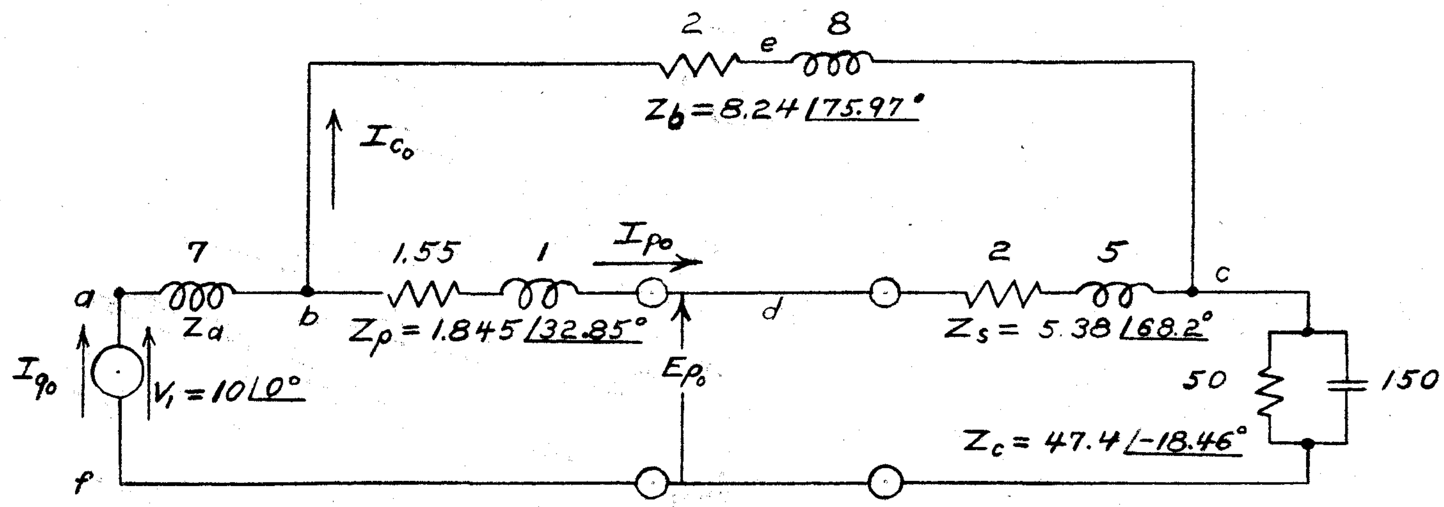

Figure 31. Base Ratio Circuit

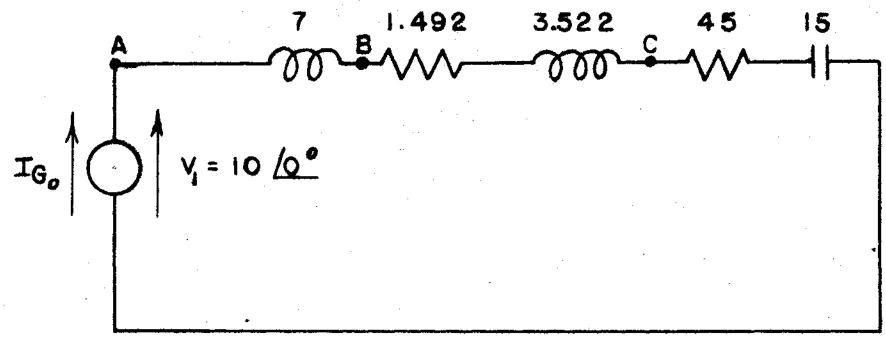

Figure 32-A. Reduction of Base Circuit

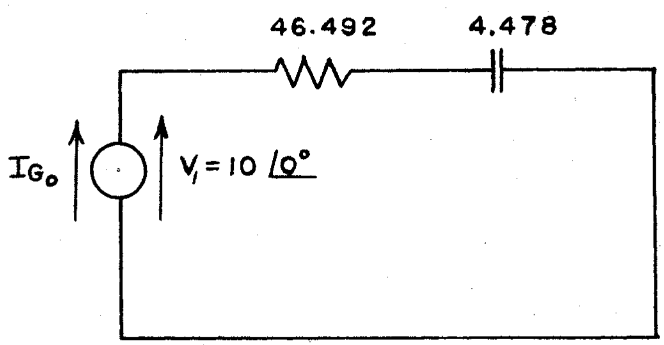

Figure 32-B. Reduced Base Circuit

The current I_{po} is,

$$I_{po} = \frac{V_{bc}}{Z_{bdc}}$$

$$= \frac{.8185\underline{/72.55}}{6.98\underline{/59.4}}$$

$$= .1173\underline{/13.15} \qquad (85)$$

$$= .1142 + j.0267 \;.$$

The current I_{co} is,

$$I_{co} = \frac{V_{bc}}{Z_{bec}}$$

$$= \frac{.8185\underline{/72.55}}{8.24\underline{/75.97}}$$

$$= .0997\underline{/-3.42} \qquad (86)$$

$$= .0996 - j.00342 \;.$$

The voltage E_{po} is,

$$E_{po} = I_{go}Z_{ef} + I_{po}Z_{dc}$$

$$= (.2145\underline{/5.53})(47.4\underline{/-18.46}) + (.1173\underline{/13.15})(5.38\underline{/68.2})$$

$$= 9.955 - j1.642$$

$$= 10.1\underline{/-9.37} \qquad (87)$$

The incremental-ratio circuit is shown in Figure 33. It must next be solved. The loop equations of the incremental ratio circuit are,

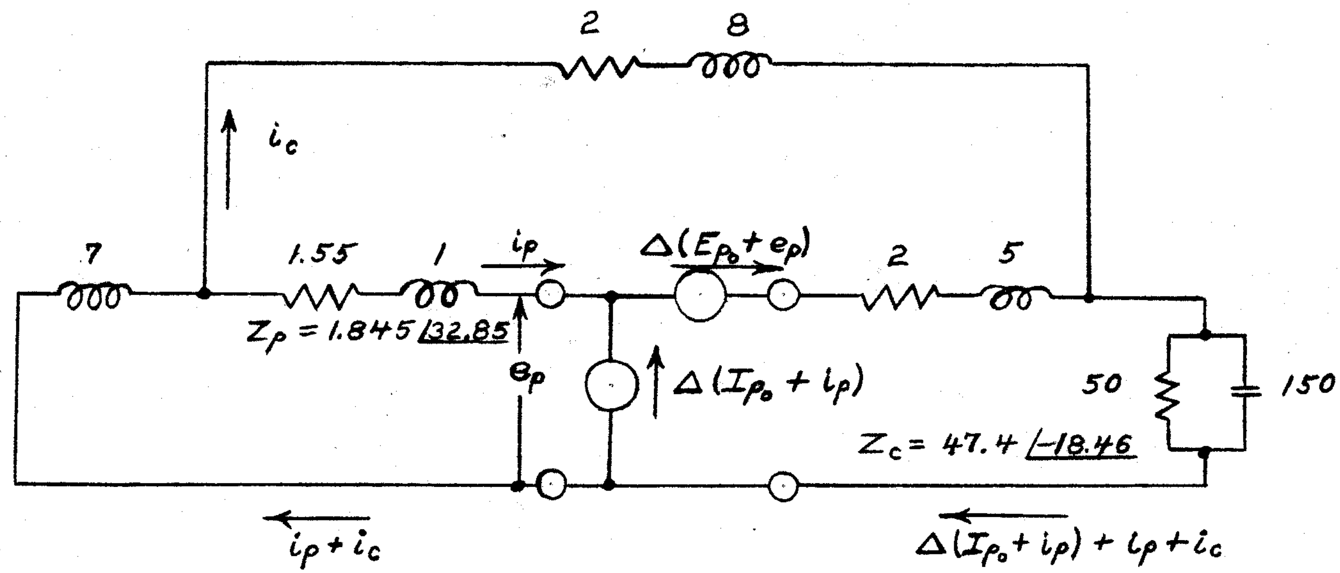

Figure 33. Incremental Ratio Circuit

$$-j7(i_c + i_p) - 1.846\underline{/32.82}\ i_p - e_p = 0 \tag{88}$$

$$e_p + \triangle(E_{po} + e_p) - 5.39\underline{/68.2}\ (\triangle(I_{po} + i_p) + i_p)$$

$$- 47.4\underline{/-18.46}\ \left[(I_{po} + i_p) + i_p + i_c \right] = 0 \tag{89}$$

$$(E_{po} + e_p) - 5.39\underline{/68.2}\ \left[\triangle(I_{po} + i_p)\right] + i_p$$

$$+ 8.24\underline{/75.97}\ i_c - 1.846\underline{/32.82}\ i_p = 0\ . \tag{90}$$

Since,

$$\triangle = A - 1 \tag{91}$$

$$= 1\underline{/10} - 1$$

$$= .1792\underline{/95} \tag{92}$$

and,

$$I_{po} = .1173\underline{/13.15} \tag{86}$$

$$E_{po} = 10.1\underline{/-9.37} \tag{87}$$

the loop equations give the simultaneous set,

$$i_p(8.14\underline{/79.03} + i_c\ (7\underline{/90}) + e_p(1) = 0 \tag{93}$$

$$i_p(-48.1\underline{/-2.02}) + i_c\ (-47.4\underline{/-18.46}) +$$

$$e_p\ (1\underline{/10}) = -.81\underline{/72.82} \tag{94}$$

$$i_p(-6.83\underline{/67.2}) + i_c\ (8.24\underline{/75.97}) +$$

$$e_p\ (.1742\underline{/95}) = -1.762\underline{/82.04} \tag{95}$$

The system determinant is,

$$D = \begin{vmatrix} 8.14\underline{/79.03} & 7\underline{/90} & 1\underline{/0} \\ -48.1\underline{/-2.02} & -47.4\underline{/-18.46} & 1\underline{/10} \\ -6.83\underline{/67.2} & 8.24\underline{/75.97} & .1742\underline{/95} \end{vmatrix} .$$

The system determinant yields,

$$D = -713\underline{/72.93} \tag{96}$$

The current i_p is,

$$i_p = \frac{\begin{vmatrix} 0 & 7\underline{/90} & 1\underline{/0} \\ -.81\underline{/72.82} & -47.4\underline{/-18.46} & 1\underline{/10} \\ -1.762\underline{/82.04} & 8.24\underline{/75.97} & .1742\underline{/95} \end{vmatrix}}{-713\underline{/72.93}}$$

$$= \frac{-81.1\underline{/76.16}}{-713/72.93}$$

$$= .1137\underline{/3.23} \tag{97}$$

$$= .113 + j.0064$$

The current i_c is,

$$i_c = \frac{\begin{vmatrix} 8.14\underline{/79.03} & 0 & 1\underline{/0} \\ -48.1\underline{/-2.02} & -.81\underline{/72.82} & 1\underline{/10} \\ -6.83\underline{/67.2} & -1.762\underline{/82.04} & .1742\underline{/95} \end{vmatrix}}{-713\underline{/72.93}} .$$

When the determinant is solved there is obtained for i_c,

$$i_c = \frac{83.3\angle 86.41}{-713\angle 72.43}$$

$$= -.1167\angle 13.48 \qquad (98)$$

$$= -.1133 - j.0272$$

The voltage e_p is,

$$e_p = \frac{\begin{vmatrix} 8.14\angle 79.03 & 7\angle 90 & 0 \\ -48.1\angle -2.02 & -47.4\angle -18.46 & -.81\angle 72.82 \\ -6.83\angle 67.2 & 8.24\angle 75.97 & -1.762\angle 82.04 \end{vmatrix}}{-713\angle 72.93}$$

$$= \frac{242.2\angle 94.37}{-713\angle 72.93}$$

$$= -.340\angle 21.44 \qquad (99)$$

$$= -.317 - j.1243$$

By superposition,

$$I_p = I_{po} + i_p \qquad (100)$$

$$= (.1142 + j.0267) + (.113 + j.0064)$$

$$= .2276 + j.0332$$

$$= .230\angle 8.3 \qquad (101)$$

$$I_c = I_{co} + i_c \tag{102}$$

$$= (.0996 - j.00594) + (-.1137 - j.0272)$$

$$= -.0141 - j.03322$$

$$= -.0362\underline{/67} \tag{103}$$

$$E_p = E_{po} + e_p \tag{104}$$

$$= (9.955 - j1.642) + (-.317 - j.1243)$$

$$= 9.638 - j1.766$$

$$= 9.82\underline{/-10.36}\ . \tag{105}$$

The current I_s is given by,

$$I_s = I_p + \quad I_p \tag{107}$$

$$= (.2276 + j.0332) + (-.0092 + j.039)$$

$$= .2184 + j.0722$$

$$= .230\underline{/18.27} \tag{108}$$

APPENDIX C. SOLUTION OF A TYPICAL LOOP CIRCUIT BY THE NETWORK ANALYZER

The loop circuit of Figure 30 was set up on the Iowa State College Network Analyzer using the phase transformer previously described to produce the required phase transformation of +10 degrees.

The phase transformer was carefully adjusted to produce a lossless transformation of +10 degrees. Readings were then taken of the vector voltages and currents in the loop circuit of Figure 30. These readings are tabulated in Table III along with the figures obtained for the same readings which have been calculated by the two-generator method and the classical method.

The circuit given as an example has been solved by three different methods. It will be observed that the values are in reasonable agreement.

TABLE III

Computed and Measured Voltages and Currents for the Circuit of Figure 28

Quantity*	Classical Method	Two-Generator Method	Network Analyzer
I_1	$0.217 \angle -0.3^\circ$	$0.214 \angle 0.0^\circ$	$0.213 \angle -1.0^\circ$
I_2	$0.207 \angle 10.98^\circ$	$0.206 \angle 10.88^\circ$	$0.204 \angle 10.12^\circ$
I_3	$-0.037 \angle 66.3^\circ$	$-0.036 \angle 67.0^\circ$	$-0.033 \angle 76.2^\circ$
I_p	$0.235 \angle 8.0^\circ$	$0.230 \angle 8.3^\circ$	$0.215 \angle 7.5^\circ$
I_c	$-0.037 \angle 66.3^\circ$	$-0.036 \angle 67.0^\circ$	$-0.033 \angle 76.2^\circ$
I_s	$0.231 \angle 18.52^\circ$	$0.230 \angle 18.27^\circ$	$0.215 \angle 17.5^\circ$
E_p		$9.82 \angle -10.36^\circ$	$9.65 \angle -10.5^\circ$
E_s		$9.84 \angle +0.3^\circ$	$9.65 \angle -0.5^\circ$

* Currents are expressed in polar amperes;
Voltages are expressed in polar volts.